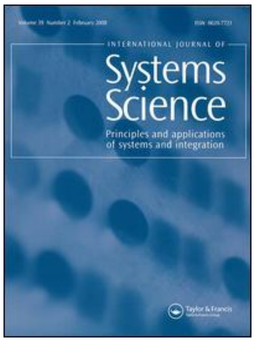

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Near-optimum steady-state regulator for discrete singularly perturbed systems with a prescribed degree of stability

QIJUN XIA†, XUEMIN SHEN†, YIQUN YING† and MING RAO†

A new approach is presented in the study of the linear-quadratic control problem of singularly perturbed discrete systems with a prescribed degree of stability. All the poles of the resulting closed-loop system are constrained to lie in a circle with the radius of $1/\alpha$. By applying a bilinear transformation, near-optimum controller design is presented in terms of continuous reduced-order problems. The stiffness problem due to the singularly perturbed system is eliminated. The method is very suitable for parallel programming. A numerical example demonstrates the efficiency of the proposed method.

1. Introduction

Linear regulator design with prescribed degree of stability has been studied in the past decade (Anderson and Moore 1969, Gopal and Ghodekar 1983, Singh *et al.* 1987) due to the advantages of the optimization procedure, which relate to sensitivity problems, the tolerance of time delay and non-linearities, etc. Because it is numerically ill-defined, the design of such optimal control for singularly perturbed system becomes computationally difficult. In this paper, we study the linear discrete singularly perturbed system with a prescribed degree of stability. Near-optimum controller design is presented in terms of reduced-order problems. The control law obtained simultaneously minimizes a quadratic-loss function and achieves closed-loop poles lying in a restricted circle of the z-plane. It is known that the main equation of optimal control theory, the Riccati equation, has a quite complicated form in the discrete-time domain. Partitioning this equation, in the spirit of singular perturbation methodology, produces a lot of terms and makes the corresponding problems numerically inefficient, even though the problem order-reduction is achieved (Gajic and Shen 1990). By applying a bilinear transformation (Kondo and Furuta 1986), the solution of the discrete algebraic Riccati equation of singularly perturbed systems is obtained by using already known results for the corresponding continuous-time algebraic Riccati equation (Gajic *et al.* 1990). The proposed method produces the reduced-order near-optimal solution, up to an arbitrary degree of accuracy $O(\epsilon^k)$, where ϵ is a small positive perturbation parameter. ($O(\epsilon^i)$ stands for $C\epsilon^i$, where C is a bounded constant and i is any arbitrary constant.) The method reduces the size of the computations required and overcomes the stiff numerical problems of large-scale systems. A numerical example demonstrates the efficiency of the proposed method.

Two main structures of discrete singularly perturbed systems were considered: the fast time scale version (Butuzov and Vasileva 1971, Blankenship 1981, Litkouhi and Khalil 1984), and slow time scale version (Phillips 1980, Naidu and Rao 1985). Since the slow time scale version presupposes the asymptotic stability of the fast

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modes, it seem that in the design procedure of stabilizing feedback controllers, the fast time scale version is very appropriate (Litkouhi and Khalil 1985). Here we will adopt the structure of Litkouhi and Khallil (1985).

2. Near-optimum regulator with a prescribed degree of stability

Consider a discrete linear singularly perturbed system (Litkouhi and Khalil 1985)

$$x_1(k+1) = (I + \epsilon A_1)x_1(k) + \epsilon A_2 x_2(k) + \epsilon B_1 u(k) \tag{1.1}$$

$$x_2(k+1) = A_3 x_1(k) + A_4 x_2(k) + B_2 u(k) \tag{1.2}$$

where $x_i \in \mathbb{R}^{ni}$, $i = 1, 2$ comprise slow and fast state vectors respectively, $u \in \mathbb{R}^m$ is the control input. A_i and B_j, $i = 1, \dots, 4, j = 1, 2$ are constant matrices of compatible dimensions and ϵ is a small positive parameter. With (1.1), (1.2), consider the performance criterion

$$J = 1/2 \sum_{k=0}^{\infty} [(\alpha)^{2k}(x^{\mathrm{T}}(k)Qx(k) + u^{\mathrm{T}}(k)Ru(k))] \tag{2}$$

where R and Q are positive-definite and positive-semi-definite matrices, respectively, and $\alpha > 1$. Defining

$$\hat{x}_1(k) = (\alpha)^k x_1(k), \quad \hat{x}_2(k) = (\alpha)^k x_2(k), \quad \hat{u}(k) = (\alpha)^k u(k) \tag{3}$$

and substituting (3) into (1) and (2), we obtain

$$\begin{aligned} \hat{x}(k+1) &= \alpha A\hat{x}(k) + \alpha B\hat{u}(k) \\ &= \hat{A}\hat{x}(k) + \hat{B}\hat{u}(k) \end{aligned} \tag{4}$$

where $\hat{A} = \alpha A$, $\hat{B} = \alpha B$.

The performance under new notation is

$$J = 1/2 \sum_{k=0}^{\infty} [(\hat{x}^{\mathrm{T}}(k)Q\hat{x}(k) + \hat{u}^{\mathrm{T}}(k)R\hat{u}(k))] \tag{5}$$

where

$$\hat{x} = \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix}, \quad Q = \begin{bmatrix} C_1^{\mathrm{T}} \\ C_2^{\mathrm{T}} \end{bmatrix} [C_1 \quad C_2] = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^{\mathrm{T}} & Q_3 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \alpha I + \alpha\epsilon A_1 & \alpha\epsilon A_2 \\ \alpha A_3 & \alpha A_4 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} \alpha\epsilon B_1 \\ \alpha B_2 \end{bmatrix} \tag{6}$$

In addition, the following condition is satisfied (Litkouhi and Khalil 1985):

$$\det(I - \alpha A_4) \neq 0 \tag{7}$$

The optimal control is given by

$$\hat{u}(k) = -(R + \hat{B}^{\mathrm{T}}P\hat{B})^{-1}\hat{B}^{\mathrm{T}}P\hat{A}\hat{x}(k) = -K\hat{x}(k) \tag{8a}$$

From (3) and (8 a), it is easy to see that

$$u(k) = -Kx(k) \tag{8b}$$

P is the unique positive-semi-definite solution of the Riccati equation

$$P = \hat{A}^{\mathrm{T}}P\hat{A} + Q - \hat{A}^{\mathrm{T}}P\hat{B}(\hat{B}^{\mathrm{T}}P\hat{B} + R)^{-1}\hat{B}^{\mathrm{T}}P\hat{A} \tag{9}$$

Due to the special structure of the problem matrices and its representation in the fast time scale, the required solution P has the form (Litkouhi and Khalil 1985)

$$P = \begin{bmatrix} P_1/\epsilon & P_2 \\ P_2^{\mathrm{T}} & P_3 \end{bmatrix} \tag{10}$$

The closed-loop form of the system (1) is given by

$$x(k+1) = [A - BK]x(k) \tag{11}$$

while the same for the system (4) is

$$\hat{x}(k+1) = \alpha[A - BK]\hat{x}(k) \tag{12}$$

The poles of the asymptotically stable (Gopal and Ghodekar 1983) closed-loop system (11) are given by the eigenvalues of $[A - BK]$ and are inside the unit circle in the complex plane, while the poles of the asymptotically stable closed-loop system (12) are given by the eigenvalues of $\alpha[A - BK]$ and are inside the circle of radius $1/\alpha$ in the complex plane. Thus α turns out to be the prescribed degree of stability. It is noted that there is an upper limit for α such that the singular perturbation structure of the discrete system (1) is preserved in the designed model represented by (4) and the power handling capacity of the input transducers does not exceed the practical limit. In order to obtain the optimal control law for the system (1) and minimize the given performance (2), and to avoid the stiffness problem of the singularly perturbed system, we solve (9) in terms of the reduced-order problem by applying a bilinear transformation.

By using a bilinear transformation (Kondo and Furuta 1986), the discrete algebraic Riccati equation (9) can be transformed into a continuous algebraic Riccati equation of the form

$$A_{\mathrm{c}}^{\mathrm{f}^{\mathrm{T}}} P + P A_{\mathrm{c}}^{\mathrm{f}} + Q_{\mathrm{c}}^{\mathrm{f}} - P S_{\mathrm{c}}^{\mathrm{f}} P = 0, \quad S_{\mathrm{c}}^{\mathrm{f}} = B_{\mathrm{c}}^{\mathrm{f}} R_{\mathrm{c}}^{\mathrm{f}^{-1}} B_{\mathrm{c}}^{\mathrm{T}} \tag{13}$$

such that the solution of (9) is equal to the solution of (13), where

$$A_{\mathrm{c}}^{\mathrm{f}} = I - 2D^{-\mathrm{T}} \tag{14a}$$

$$S_{\mathrm{c}}^{\mathrm{f}} = 2(I + \hat{A})^{-1} S_{\mathrm{d}} D^{-1}, \quad S_{\mathrm{d}} = \hat{B} R^{-1} \hat{B}^{\mathrm{T}} \tag{14b}$$

$$Q_{\mathrm{c}}^{\mathrm{f}} = 2D^{-1} Q (I + \hat{A})^{-1} \tag{14c}$$

$$D = (I + \hat{A}^{\mathrm{T}}) + Q(I + \hat{A})^{-1} S_{\mathrm{d}} \tag{14d}$$

assuming that $(I + \hat{A})^{-1}$ exists. Note that a superscript f stands for the fast time scale representation. It can be easily seen that the matrix

$$I + \hat{A} = \begin{bmatrix} (1+\alpha)I + \alpha\epsilon A_1 & \alpha\epsilon A_2 \\ \alpha A_3 & I + \alpha A_4 \end{bmatrix} \tag{15}$$

is invertible for small values of ϵ if and only if the matrix $(I + \alpha A_4)$ is invertible. This condition is satisfied under the assumption given in (7). It is important to point out that the matrix D defined in (14d) is non-singular if (7) holds (Bar-Ness and Halbersberg 1980).

Since there is no difference in the use of either the slow or fast time scale representation for the continuous-time LQ control problem of singularly perturbed systems, we adopt the slow time scale version for this problem (it is customary to represent continuous time singularly perturbed systems by their slow time version (Kokotovic and Khalil 1986, Kokotovic *et al.* 1986).

The slow time version of (14) can be obtained by multiplying the matrix A_c^f by $1/\epsilon$ and matrix S_c^f by $1/\epsilon^2$. Introducing a notation for the compatible partition of these matrices, we have

$$A_c = \begin{bmatrix} A_{11} & A_{12} \\ A_{21}/\epsilon & A_{22}/\epsilon \end{bmatrix}, \quad S_c = \begin{bmatrix} S_{11} & S_{12}/\epsilon \\ S_{12}^T/\epsilon & S_{22}/\epsilon^2 \end{bmatrix} \tag{16}$$

By doing this, the required solution P (9), will be multiplied by ϵ, that is

$$\epsilon P = P_c = \begin{bmatrix} P_1 & \epsilon P_2 \\ \epsilon P_2^T & \epsilon P_3 \end{bmatrix} \tag{17}$$

Going from the fast time version to the slow time version does not change the matrix Q_c. It is partitioned as

$$Q_c = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} = Q_c^f \tag{18}$$

Note that the partitions defined in (16)–(18) have to be performed by a computer only, in the process of the calculations, and there is no need for the corresponding analytical expressions.

The solution of (13) can be found in terms of the reduced-order problems by imposing standard stabilizability-detectability assumptions on the slow and fast subsystems. An efficient recursive reduced-order algorithm for solving (13) is obtained by Gajic (1986) and Gajic *et al.* (1990). The algorithm is briefly summarized here taking into account the specific features of the problem under study.

First, we derive expressions for B_c and R_c so that the analogy between the discrete quantities $(\hat{A}, \hat{B}, Q, R)$ and continuous ones (A_c, B_c, Q_c, R_c) is completed. By definition

$$S_c^f = B_c^f R_c^{f^{-1}} B_c^{f^T} \tag{19}$$

From (14 *b*), we have

$$S_c^f = 2(I+\hat{A})^{-1} S_d D^{-1} (I+\hat{A}^T)(I+\hat{A}^T)^{-1}$$

Since

$$\begin{aligned} S_d D^{-1}(I+\hat{A}^T) &= S_d[(I+\hat{A}^T)^{-1} D]^{-1} \\ &= S_d[I + (I+\hat{A}^T)^{-1} Q (I+\hat{A})^{-1} S_d]^{-1} \\ &= \hat{B}[R + \hat{B}^T (I+\hat{A}^T)^{-1} Q (I+\hat{A})^{-1} \hat{B}]^{-1} \hat{B}^T \end{aligned}$$

(the last step in this expression is justified in Bar-Ness and Halbersberg 1980, we obtain

$$S_c^f = 2(I+\hat{A})^{-1}\hat{B}[R + \hat{B}^T(I+\hat{A}^T)^{-1} Q (I+\hat{A})^{-1}\hat{B}]^{-1}\hat{B}^T(I+\hat{A}^T)^{-1} \tag{20}$$

Comparing (19) and (20), we conclude that

$$B_c^f = (I+\hat{A})^{-1}\hat{B} = \begin{bmatrix} \epsilon B_1^f \\ B_2^f \end{bmatrix} \tag{21}$$

and

$$R_c^f = 0{\cdot}5[R + \hat{B}^T(I+\hat{A}^T)^{-1} Q (I+\hat{A})^{-1}\hat{B}] = R_c \tag{22}$$

Note that R_c is positive-definite. The slow time version of (21) is

$$B_c = (1/\epsilon)B_c^f = \begin{bmatrix} B_1^f \\ B_2^f/\epsilon \end{bmatrix} \tag{23}$$

The $O(\epsilon)$ approximation of (13) subject to (16), (18) and (22)–(23) can be obtained from the following reduced-order set of algebraic equations (Gajic 1986, Gajic *et al.* 1990):

$$0 = \mathbf{P}_1\mathbf{A} + \mathbf{A}^T\mathbf{P}_1 + \mathbf{Q} - \mathbf{P}_1\mathbf{S}\mathbf{P}_1 \tag{24a}$$

$$0 = \mathbf{P}_3A_{22} + A_{22}^T\mathbf{P}_3 + Q_{22} - \mathbf{P}_3S_{22}\mathbf{P}_3 \tag{24b}$$

$$\mathbf{P}_2 = \mathbf{P}_1Z_1 - Z_2 \tag{24c}$$

where newly defined matrices are given by

$$\mathbf{A} = A_0 - B_0R_0^{-1}r^TQ_0, \quad A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}, \quad B_0 = B_1^f - A_{12}A_{22}^{-1}B_2^f$$

$$R_0 = R_c + r^Tr, \quad r = C_2A_{22}^{-1}B_2^f, \quad \mathbf{S} = B_0R_0^{-1}B_0^T, \quad S_{22} = B_2^fR_c^{-1}B_2^{T^f}$$

$$Q_0 = C_1 - C_2A_{22}^{-1}A_{21}, \quad Q_{11} = C_1^TC_1, \quad Q_{22} = C_2^TC_2, \quad \mathbf{Q} = Q_0^T(I - rR_0^{-1}r^T)Q_0$$

$$Z_1 = (S_{12}\mathbf{P}_3 - A_{12})(A_{22} - S_{22}\mathbf{P}_3)^{-1}, \quad Z_2 = (A_3^T\mathbf{P}_3 + Q_{12})(A_{22} - S_{22}\mathbf{P}_3)^{-1}$$

The unique positive-semi-definite stabilizing solution of (24) exists under the following assumption.

Assumption

The triples $(\mathbf{A}, B_0, \sqrt{\mathbf{Q}})$ and $(A_{22}, B_2^f, \sqrt{Q_{22}})$ are stabilizable-detectable.

Defining the approximation errors as

$$P_i = \mathbf{P}_i + \epsilon E_i, \quad i = 1, 2, 3 \tag{25}$$

the recursive reduced-order algorithm, with the rate of convergence of $O(\epsilon)$, was derived by Gajic (1986) and Gajic *et al.* (1990).

$$E_1^{(j+1)}D_1 + D_1^TE_1^{(j+1)} = D^TH_1^{(j)^T} + H_1^{(j)}D + D^TH_3^{(j)}D + \epsilon H_2^{(j)} \tag{26a}$$

$$E_2^{(j+1)}D_3 + E_1^{(j+1)}D_{21} + D_{22}^TE_3^{(j+1)} = -H_1^{(j,j+1)} \tag{26b}$$

$$E_3^{(j+1)}D_3 + D_3^TE_3^{(j+1)} = H_3^{(j)} \tag{26c}$$

with $j = 0, 1, 2, \ldots$, and $E_1^{(0)} = 0$, $E_2^{(0)} = 0$, $E_3^{(0)} = 0$, where newly defined matrices are given as

$$D_1 = A_{11} - S_{11}\mathbf{P}_1 - S_{12}\mathbf{P}_2^T - D_{21}D_3^{-1}D_{22} = D_{11} - D_{21}D_3^{-1}D_{22}$$

$$D_3 = A_{22} - S_{22}\mathbf{P}_3, \quad D = D_3^{-1}D_{22}$$

$$D_{21} = A_{12} - S_{12}\mathbf{P}_3, \quad D_{22} = A_{21} - S_{12}^T\mathbf{P}_1 - S_{22}\mathbf{P}_2^T$$

$$H_1^{(j,j+1)} = A_{11}^TP_2^{(j)} - P_1^{(j+1)}S_{11}P_2^{(j)} - P_2^{(j)}S_{12}^TP_3^{(j)} - \epsilon(E_1^{(j+1)}S_{12}E_3^{(j+1)} + E_2^{(j)}S_{22}E_2^{(j)})$$

$$H_2^{(j)} = E_1^{(j)} S_{11} E_1^{(j)} + E_1^{(j)} S_{12} E_2^{(j)^{\mathrm{T}}} + E_2^{(j)} S_{12}^{\mathrm{T}} E_1^{(j)} + E_2^{(j)} S_{22} E_2^{(j)^{\mathrm{T}}}$$

$$H_3^{(j)} = -P_2^{(j)^{\mathrm{T}}} A_{12} - A_{12}^{\mathrm{T}} P_2^{(j)} + \epsilon P_2^{(j)^{\mathrm{T}}} S_{11} P_2^{(j)} + \epsilon E_3^{(j)} S_{22} E_3^{(j)}$$
$$+ P_2^{(j)^{\mathrm{T}}} S_{12} P_3^{(j)} + P_3^{(j)} S_{12}^{\mathrm{T}} P_2^{(j)}$$

It is important to point out that D_1 and D_3 are stable matrices (Gajic 1986).

The rate of convergence of (26) is $O(\epsilon)$, that is

$$\|P_i - P_i^{(j)}\| = O(\epsilon^j), \quad i = 1, 2, 3, \ j = 0, 1, 2, \ldots \tag{27}$$

where

$$P_i^{(j)} = \mathbf{P}_i + \epsilon E_i^{(j)}, \quad i = 1, 2, 3, \ j = 0, 1, 2, \ldots \tag{28}$$

The proposed algorithm for the reduced-order solution of a singularly perturbed discrete algebraic Riccati equation has the following form:

(*a*) transform (9) into (13) by using the bilinear transformation defined in (14);

(*b*) solve (13) by using the recursive reduced-order algorithm defined by (24)–(28).

Equations (24)–(28) can be implemented by using ideas from parallel programming ((26 *a*) and (26 *c*) can be solved simultaneously) which will play an important role in the real-time control. After obtaining the solution of the Riccati equation $P^{(j)}$, the near-optimum control (8 *a*) is

$$\hat{u}^{(j)}(k) = -(R + \hat{B}^{\mathrm{T}} P^{(j)} \hat{B})^{-1} \hat{B}^{\mathrm{T}} P^{(j)} \hat{A} \hat{x}(k) \tag{29}$$

So far, we have developed a very efficient technique for generating $P^{(j)}$ by using the recursive reduced-order scheme (24)–(28), such that each iteration improves the accuracy by an order of magnitude (see (27)). Thus, the proposed algorithm, the theoretical results obtained by Litkouhi and Khalil (1984) and given in (29) comprise a new method for solving the linear-quadratic optimal control problem of singularly perturbed discrete systems with a prescribed degree of stability. The efficiency of this method is demonstrated on a real world example in the next section.

3. Numerical example

A fifth-order discrete model of a steam power system (Mahmoud 1982) has a singularly perturbed discrete form, with three fast and two slow variables. The problem matrices A and B are given by

$$A = \begin{bmatrix} 0.9150 & 0.0510 & 0.0380 & 0.0150 & 0.0380 \\ -0.0300 & 0.8990 & -0.0005 & 0.0460 & 0.1110 \\ -0.0060 & 0.4680 & 0.2470 & 0.0140 & 0.0480 \\ -0.7150 & -0.0220 & -0.0211 & 0.2400 & -0.0240 \\ -0.1480 & -0.0030 & -0.0040 & 0.0900 & 0.0260 \end{bmatrix}$$

$$B^{\mathrm{T}} = [0.0098 \quad 0.1220 \quad 0.0360 \quad 0.5620 \quad 0.1150]$$

The linear-quadratic optimal control problem is solved for weighting matrices $R = I$, $Q = I_5$ and $\epsilon = 0.264$.

The eigenvalues of the matrix A_4 are $0{\cdot}2507 \pm j0{\cdot}0254$, $0{\cdot}0298$, so that (7) is satisfied.

Simulation results for the discrete algebraic Riccati equation open-loop and closed-loop eigenvalues are presented in Tables 1 and 2.

Table 1 shows that we have quite rapid convergence for the optimal solution of the Riccati equation by using the reduced-order algorithm, namely, it justifies the result of (27) because $(0{\cdot}264)^6 = 3{\cdot}386 \times 10^{-4}$. The problem of ill-conditioning due to the singularly perturbed system is eliminated.

From Table 2, we can see that all the poles of the resulting closed-loop system are constrained to lie in a circle with the radius of $1/\alpha$, even for the open-loop unstable

	j	P_1		P_2			P_3		
$\alpha = 1{\cdot}0$	0	7·9952	1·3082	1·5172	0·1277	1·7076	4·0360	−0·0025	0·0221
		1·3082	5·2683	0·9179	1·0555	2·4192	−0·0025	3·9927	0·0061
							0·0221	0·0061	3·8076
	1	8·2317	1·4922	1·4205	0·0852	1·6496	4·1154	0·0389	0·1594
		1·4922	5·4185	0·8919	0·9155	2·3133	0·0389	4·0521	0·1500
							0·1594	0·1500	4·1640
	2	8·2059	1·4669	1·4256	0·0909	1·6474	4·1092	0·0348	0·1514
		1·4669	5·3914	0·8880	0·9333	2·3131	0·0348	4·0429	0·1331
							0·1514	0·1331	4·1432
	3	8·2083	1·4699	1·4255	0·0903	1·6483	4·1096	0·0351	0·1515
		1·4699	5·3953	0·8892	0·9312	2·3143	0·0351	4·0441	0·1349
							0·1515	0·1349	4·1439
	4†	8·2081	1·4696	1·4255	0·0904	1·6481	4·1096	0·0351	0·1516
		1·4696	5·3948	0·8890	0·9315	2·3140	0·0351	4·0439	0·1348
							0·1516	0·1348	4·1440
$\alpha = 1{\cdot}1$	0	22·2131	4·7145	4·7245	1·9431	5·6207	4·0920	−0·0027	0·0252
		4·7145	8·8714	1·8183	2·0969	4·8248	−0·0027	4·0247	0·0073
							0·0252	0·0073	3·8099
	1	24·3377	5·7671	4·8480	1·8784	5·8882	4·3596	0·1487	0·4044
		5·7671	9·5878	1·9598	1·9876	4·9321	0·1487	4·1773	0·3611
							0·4044	0·3611	4·6418
	2	24·2980	5·7326	4·8236	1·8785	5·8555	4·3555	0·1401	0·4042
		5·7326	9·5407	1·9328	1·9888	4·9012	0·1401	4·1591	0·3347
							0·4042	0·3347	4·6277
	3	24·3006	5·7327	4·8271	1·8789	5·8584	4·3549	0·1403	0·4023
		5·7327	9·5434	1·9368	1·9901	4·9056	0·1403	4·1607	0·3361
							0·4023	0·3361	4·6256
	4	24·3008	5·7333	4·8267	1·8788	5·8583	4·3551	0·1404	0·4027
		5·7333	9·5435	1·9363	1·9898	4·9052	0·1404	4·1607	0·3362
							0·4027	0·3362	4·6261
	5†	24·3007	5·7332	4·8267	1·8788	5·8583	4·3551	0·1404	0·4027
		5·7332	9·5435	1·9363	1·9899	4·9052	0·1404	4·1606	0·3362
							0·4027	0·3362	4·6261

† Exact solutions.

Table 1. Reduced-order solution of the discrete algebraic Riccati equation for different α.

α	Eigenvalues: Open-loop	Eigenvalues: Closed-loop	$1/\alpha$
1·00	0·8929 + 0·0943i 0·8929 − 0·0943i 0·2507 + 0·0254i 0·2507 − 0·0254i 0·0298	0·8463 + 0·0375i 0·8463 − 0·0375i 0·0301 0·1936 0·2433	1·00
1·05	0·9376 + 0·0990i 0·9376 + 0·0990i 0·2632 + 0·0267i 0·2632 − 0·0267i 0·0313	0·8247 + 0·0267i 0·8247 − 0·0267i 0·0302 0·1875 0·2445	0·9524
1·10	0·9822 + 0·1038i 0·9822 − 0·1038i 0·2757 + 0·0280i 0·2757 − 0·0280i 0·0328	0·7935 + 0·0071i 0·7935 − 0·0071i 0·0302 0·01816 0·2455	0·9091
1·15	1·0269 + 0·1085i 1·0269 − 0·1085i 0·2883 + 0·0293i 0·2883 − 0·0293i 0·0343	0·7751 0·7314 0·1759 0·0320 0·2465	0·8696
1·20	1·0715 + 0·1132i 1·0715 − 0·1132i 0·3008 + 0·03505i 0·3008 − 0·03505i 0·0357	0·7366 0·6789 0·1703 0·0303 0·2473	0·8333
1·25	1·1162 + 0·1179i 1·1162 − 0·1179i 0·3133 + 0·0318i 0·3133 − 0·0318i 0·0372	0·6941 0·6289 0·1649 0·0303 0·2482	0·8000

Table 2. Open-loop and closed-loop eigenvalues for different α.

system. As a result, there is inherently a great margin for tolerance of time delay, non-linearity and system parameter variations (Anderson and Moore 1969).

4. Conclusions

The discrete linear-quadratic control problem with prescribed degree of stability is completely and efficiently solved (up to an arbitrary order of accuracy) in terms of the reduced-order problems. The proposed algorithm has a parallel structure and is very convenient for real-time implementation.

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