PHY-Aware Distributed Scheduling for Ad Hoc Communications with Physical Interference Model

Weiyan Ge, Junshan Zhang, Jeffrey E. Wieselthier, and Xuemin (Sherman) Shen

***Abstract*—We consider a random-access-based ad hoc network, where different links use mini-slots to contend for the channel, and then successful links transmit data packets, as in CSMA. The focus of our study is to develop optimal strategies for physical-layer-aware (PHY-aware) distributed scheduling, which involves a joint process of channel probing and distributed scheduling. Because of channel fading and cochannel interference, the *signal-to-interference-plus-noise-ratio* (SINR) across links is highly dynamic and can exhibit significant variation. In the low SINR case, further channel probing is likely to lead to better SINR conditions and hence yield higher throughput. The desired tradeoff boils down to judiciously choosing the optimal stopping strategy for channel probing before data transmissions.**

In this paper, we investigate PHY-aware distributed scheduling, aiming to maximize the overall network throughput. The problem under consideration is inherently challenging: 1) multiple links can transmit successfully simultaneously and the number of simultaneously transmitting links is random; and 2) the network throughput is the sum rate of all transmitting links, but each link involved in the transmission has no knowledge of the instantaneous rates of other links, and the stopping decision is made in a distributed manner based on local information only. We use optimal stopping theory to tackle this challenge, and show that the optimal policy for distributed scheduling has a threshold structure. Accordingly, after a channel probing, a link would proceed with data transmissions only if a function of its instantaneous rate is greater than the optimal rate threshold. Observing that the network throughput depends heavily on the contention probability of each link, we generalize the study to jointly optimize the rate threshold and the contention probability, and propose a two-stage algorithm for computing the pair of optimal rate threshold and contention probability by using fractional optimization and geometric programming.

***Index Terms*—Distributed scheduling, physical interference model, optimal stopping, ad hoc communications.**

I. INTRODUCTION

RECENTLY, there has been an increasing interest in PHY-aware scheduling to improve spectrum efficiency by exploiting rich diversities inherent in wireless communications. Most existing studies along this line assume that the scheduler has knowledge of the instantaneous channel conditions for all links (transmitter/receiver pairs), and therefore the scheduling is centralized (see [12], [13], [18], [19] and the references therein). However, in an ad hoc network, the communications are peer-to-peer, and each link has no knowledge of other links' channel conditions, which makes it very challenging to carry out PHY-aware scheduling.

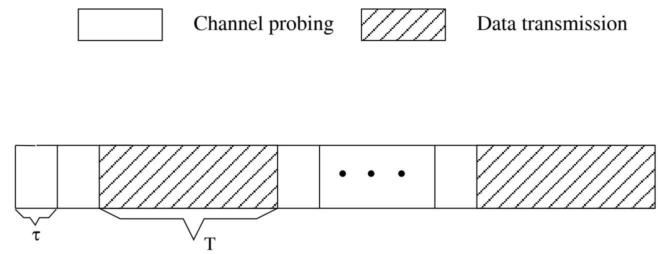


Fig. 1. A sample realization of channel probing and data transmission.

In this paper, we consider a random-access-based ad hoc network under the physical interference model, where links contend for the channel using mini-slots. Our principal goal is to maximize the average network throughput. When a link is successful in this contention process, its data (which is much greater in size than the probing packet) is then transmitted, as illustrated in Fig. 1. Before data transmissions, each transmitter sends probing signals in a mini-slot (of duration τ) with specific probability, and its corresponding receiver can estimate the signal-to-interference-plus-noise-ratio (SINR) and predict the supportable transmission rate [21]. It would proceed with data transmission over duration T[1] only if the predicted rate is high enough (indicating a "good" SINR condition); otherwise, it may skip the transmission, in the hope that further channel probing may lead to better SINR conditions and hence yield higher throughput. If no contending links transmit after the first probing (as determined distributedly by monitoring the channel, as in CSMA), all links would probe the channel again using another mini-slot. The probing process continues until one or more links with good channel conditions transmit data over the channel.

Our study here is built on the initial steps on developing distributed opportunistic scheduling (DOS) under the collision model [20], which can be summarized as follows. Consider a single-hop ad hoc network. After a successful channel contention, the corresponding successful link may skip the data transmission if the observed channel condition is "poor". All the links re-contend for the channel, in the hope that some link with better channel conditions can transmit after the re-contention. In this way, multiuser diversity across links

Manuscript received June 17, 2008; revised December 22, 2008; accepted January 1, 2009. The associate editor coordinating the review of this paper and approving it for publication was Y. Fang.

W. Ge is with Qualcomm Inc. San Diego, CA, 92121, USA (e-mail: wge@qualcomm.com).

J. Zhang is with the Department of Electrical Engineering, Arizona State University, Tempe, AZ, 85287, USA (e-mail: junshan.zhang@asu.edu).

J. E. Wieselthier is currently with Wieselthier Research, Silver Spring, MD 20901, USA (e-mail: jeff@wieselthier.com).

X. (S.) Shen is with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada (e-mail: xshen@bbcr.uwaterloo.ca).

This research is supported in part by the National Science Foundation through the grants ANI-0238550 and CNS-0721820.

Digital Object Identifier 10.1109/TWC.2008.080798

[1]Throughout this paper, we assume T is fixed, and is no greater than the channel coherence time.

1536-1276/09$25.00 © 2009 IEEE

and time diversity across slots can be exploited in a joint manner. On the other hand, each channel probing comes with a cost in terms of the contention time. Clearly, there is a *tradeoff* between the throughput gain from better channel conditions and the cost for further channel probing. The desired tradeoff boils down to judiciously choosing the optimal stopping strategy for channel probing.

Note that in [20] all links contend for the channel based on the collision model, which assumes that a channel contention by a link is successful only if no other links transmit at the same time. However, there has recently been a general consensus that the collision model is inadequate to characterize the probabilistic receptions in wireless communications. In fact, with the advent of new signal processing techniques, such as multiuser detection, spread spectrum, and space-time processing, it is possible to simultaneously decode multiple packets even when a "collision" happens. These new techniques call for an SINR-based reception model, known as the *physical interference model*. Under this new model, a transmission is said to be successful if its SINR is greater than a pre-determined threshold. The new reception model at the PHY layer opens great opportunities for designing new MAC protocols in wireless networks.

The problem of PHY-aware distributed scheduling with physical interference is in general difficult. One unique challenge is that multiple links can transmit successfully through one common channel; furthermore, each link has to make the decision to transmit or not based on local information only, because links involved in the transmission have no knowledge of the instantaneous transmission rates of other links, but the network throughput depends on the data rates of all transmitting links. Moreover, the number of simultaneously transmitting links is random, and heavily depends on the contention probability of each link. Roughly speaking, a larger contention probability would increase the number of probing links and thus incur stronger cochannel interference. On the other hand, a smaller contention probability would reduce the number of links participating in the transmission. Clearly, the overall network throughput is low at both the two extremes and one would expect that the maximum lies somewhere in between. In summary, PHY-aware distributed scheduling under the physical interference model requires that each link makes its own decision to transmit or not in the presence of a number of uncertainties, namely the SINR condition, the number of contending links, and the number of minislots required for channel probing. Consequently, this is a challenging problem.

In this paper, we study PHY-aware distributed scheduling with physical interference from a network-centric perspective, with the objective being to maximize the network throughput. Appealing to optimal stopping theory, we model the problem of choosing the optimal stopping rule for channel probing and data transmissions as a *maximal rate of return* problem. We investigate thoroughly the optimal strategy for PHY-aware distributed scheduling, where each link makes the stopping decision independently based on local information. We show that the optimal stopping rule for channel probing and data transmission is threshold-based. Accordingly, after a channel probing, a link would proceed with data transmission only if a function (to be determined in Section III) of its instantaneous transmission rate is greater than the optimal rate threshold. It is worth noting that the function is different for each individual link, but the rate threshold is the same across different links. Furthermore, we develop algorithms to compute the optimal rate threshold. For the case with fixed contention probability, exhaustive search can be used to find the optimal rate threshold. Needless to say, it is much more involved to tackle the problem of jointly optimizing the rate threshold and the contention probability. We propose a two-stage iterative algorithm for computing the optimal threshold-probability pair by using fractional optimization and geometric programming. We also take a closer look at the homogeneous case where all links have the same channel statistics. In this special case, the optimal rate threshold and contention probability are the same for all links. It turns out that the objective function is a quasi-concave function, and the pair of optimal rate threshold and contention probability can be computed accordingly by using the bi-section method [4]. To improve the network performance, we also propose an enhanced scheme, in which each link makes its decision based on both its own channel information and available partial information about other links' decisions.

In related work, there has been much interest in MAC scheduling under the physical interference model. In [11], the scheduling of wireless links under the physical interference model was proved to be NP-complete, and an approximation algorithm based on graph theory was proposed. The study in [5] took an alternative approach by using a greedy method to compute a suboptimal schedule with a proven approximation factor. In [7], an optimization problem was formulated which jointly decides scheduling and MIMO stream allocation in order to maximize the average sum link rate in a single-hop network. A multipacket reception (MPR) model was proposed in [2], and the capture probability and local throughput of MPR in random access networks were investigated in [17], [16]. In [9], joint power control and centralized link scheduling under physical interference was characterized. In [15], the optimal scheduling problem for physical-interference-model networks was formulated as a non-cooperative game, where each link computes its optimal transmission policy to maximize its own utility. We note that some of the above studies (e.g., [5], [7], [9], [11]) consider centralized scheduling, while the others (e.g., [2], [15], [17]) assume many-to-one Aloha schemes, where the central controller (or common receiver) can "coordinate" the transmissions among different links, based on the channel information of all links. In contrast, it remains under-explored to develop PHY-aware distributed scheduling for ad hoc (peer-to-peer) communications under physical interference. To fill the void, this paper focuses on ad hoc communications assuming no centralized coordination, and transmission scheduling is done distributedly with local information. Moreover, the transmitter nodes have no knowledge of other links' channel conditions, and even their own channel conditions are not available before channel probing.

The remainder of the paper is organized as follows. In Section II, we introduce the system model, and give the background results on distributed opportunistic scheduling under the collision model. In Section III, we present in

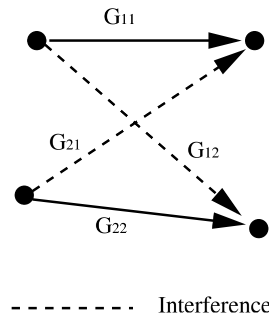


Fig. 2. A single-hop ad hoc network.

depth PHY-aware distributed scheduling under the physical interference model. Section IV focuses on a special case with a symmetric channel. In Section V, we propose an enhanced scheme to improve further the performance by allowing link collaboration. In Section VI, numerical examples are presented to corroborate the theoretic results. Finally, we present our conclusions in Section VII.

II. System Model and Background

A. System Model

We consider a single-hop ad hoc network with M links, where at each mini-slot, each link contends for the channel with probability $p_i, i = 1, 2, \cdots, M$ [18], [21]. Let S denote a non-empty set of links, and $\Pr(S)$ denote the probability that links in set S contend for the channel. Then, as in [14],

$$Pr(S) = \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j). \tag{1}$$

Under the physical interference model, the probability that a packet is received successfully (possibly in the presence of other transmissions) depends on its own channel condition as well as the strength of the cochannel interference. Specifically, let $s_i = 1$ indicate that link i transmits a data packet, and $s_i = 0$ for no data transmission. Let $r_i = 1$ indicate a successful reception of the data packet for link i, and $r_i = 0$ otherwise. Let K be a set of links such that $K \subseteq S$, and $q_{S,K}$ be the probability that links in set K transmit successfully given that links in set S contend for the channel. It follows that

$$q_{S,K} = Pr(r_{i \in K} = 1, r_{i \notin K} = 0 | s_{i \in S} = 1, s_{i \notin S} = 0). \tag{2}$$

Let $\mathcal{M}$ denote the set of all links, i.e., $\mathcal{M} = \{1, 2, \cdots, M\}$. The set of conditional probability $\mathcal{P} = \{q_{S,K}, S \subseteq 2^{\mathcal{M}}, K \subseteq S\}$ completely specifies the probability space for the physical interference model [14].

Let P denote the transmission power, G_{ij} denote the channel gain from the ith transmitter to the jth receiver, as shown in Fig. 2, and $\mathbf{G} = \{G_{ij}\}$ be the $M \times M$ channel gain matrix. The SINR for link i is given by

$$\mathrm{SINR_i} \triangleq \frac{G_{ii} P}{\sum_{j \neq i} G_{ji} P + \eta_i}, \tag{3}$$

where η_i is the power of thermal noise at the receiver of link i.

It is clear that the transmission rate is an increasing function of SINR. In practical systems, continuous control of transmission rate may not be possible, and the rates are often quantized to discrete values. For instance, in IEEE 802.11b, the transmission rate is a function of SINR and can be 1Mbps, 2Mbps, 5.5Mbps and 11Mbps. Generally, let $\{R^l, l = 1, 2, \cdots, L\}$ denote the available discrete transmission rates, with $0 < R^1 < R^2 < \cdots < R^L$, and the corresponding rate distribution is given by

$$R = \begin{cases} 0, & \text{if } \ \mathrm{SINR} < \gamma_1, \\ R^l, & \text{if } \ \gamma_l \leq \mathrm{SINR} < \gamma_{l+1}, \quad l = 1, 2, \cdots, L-1, \\ R^L, & \text{if } \ \mathrm{SINR} \geq \gamma_L, \end{cases}$$

where $\gamma_1, \gamma_2, \cdots, \gamma_L$ are the SINR thresholds predetermined by the communication system.

B. Background on Distributed Opportunistic Scheduling: The Collision Channel Model

In [20], we have taken some initial steps to study distributed opportunistic scheduling (DOS) under the collision model, where the channel probing is successful only if one link contends for the channel. Specifically, we have shown that the scheduling problem can be cast as a maximal rate of return problem in optimal stopping theory [6], [8], where the rate of return is the average network throughput, x, which can be determined by the stopping time N as

$$x = \frac{E[R_N T]}{E[T_N]}, \tag{4}$$

where R_N is the instantaneous data rate at the N-th successful channel contention (or channel probing), and T_N is the total time that includes the contention time and the data transmission time. Then, the scheduling problem boils down to finding the optimal stopping policy N^* that maximizes the average network throughput, i.e.,

$$N^* \triangleq \arg\max_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \tag{5}$$

where

$$Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \tag{6}$$

Assuming that the rates $\{R_n, n = 1, 2, \cdots\}$ are independent, and with finite second moments, the following result follows directly from [20].

Lemma 2.1: a) The optimal stopping rule N^* for distributed opportunistic scheduling exists, and is given by

$$N^* = \min\{n \geq 1 : R_n \geq x^*\}. \tag{7}$$

The maximum throughput x^* in (7) is an optimal threshold, and is given by

$$x^* = \sum_{l=1}^{L} \frac{\sum_{i=l}^{L} R^i p^i}{\delta/q_1 + \sum_{i=l}^{L} p^i} \mathbf{I} \left(R^{l-1} \leq \frac{\sum_{i=l}^{L} R^i p^i}{\delta/q_1 + \sum_{i=l}^{L} p_i} \leq R^l \right),$$

where $\delta = \tau/T$, $R^0 \triangleq 0$, $p^i \triangleq \Pr\{R_n = R^i\}$, $\mathbf{I}(\cdot)$ is the indicator function, and q_1 is the probability that exactly one link contend for the channel, which is given by

$$q_1 = \sum_{i=1}^{M} p_i \prod_{j \neq i} (1 - p_j). \tag{8}$$

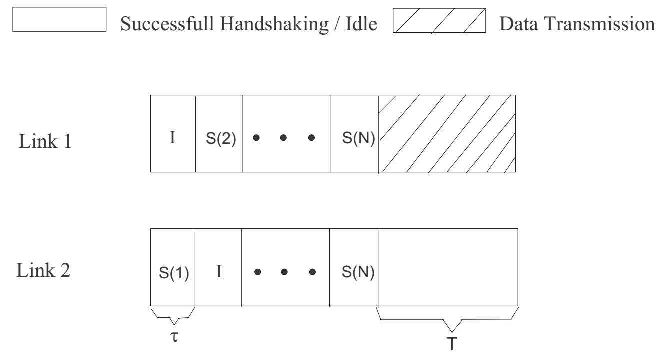


Fig. 3. A sample realization of channel probing and data transmission for a two-link network.

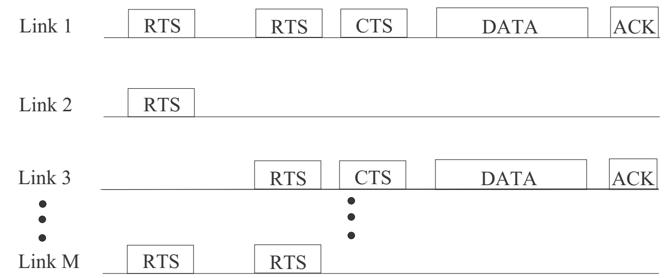


Fig. 4. An example of the RTS-CTS handshaking for the network with physical interference.

III. PHY-Aware Distributed Scheduling with Physical Interference

In this section, we investigate PHY-aware distributed scheduling under the physical interference model. Let S_n denote the set of links that probe the channels at the nth probing, and K_n denote the subset of links with "good" channel conditions; therefore, $K_n \subseteq S_n$. Let $R_{i,n}$ denote the rate of link i at the nth probing. Since $R_{i,n}$ depends on the time varying channel condition and the strength of the cochannel interference, it is random.

Under the physical interference model, multiple links can transmit successfully simultaneously, and this is in contrast to the study based on the collision model where at each time only one link can transmit successfully over the channel. The total rate would be the sum of the rates of all successful links that participate in the transmission. However, in ad hoc networks, each link has no knowledge of the channel state information of other links, and has to make its decision based on local information only, which makes the problem inherently challenging. In the following, we characterize the optimal stopping rule and the corresponding network throughput.

A. An Example

To illustrate the basic idea of PHY-aware distributed scheduling, we depict in Fig. 3 an example of an ad hoc network with two links. During the first mini-slot (which has a duration of τ), only link 2 probes the channel by sending an RTS packet, but the rate $R_{2,1}$ is small (indicating "poor" channel condition). It then gives up the transmission opportunity. At the next mini-slot, the same thing happens to link 1, and it also gives up the transmission. The process continues until the Nth mini-slot, in which both links probe the channel, and $R_{1,N}$ is big but $R_{2,N}$ is small. As a result, link 1 would transmit with duration T and rate $R_{1,N}$. To avoid impairing the ongoing transmissions, other links are not permitted to probe the channel when some links are transmitting. As a consequence, link 2 would keep silent for a duration T.

The distributed optimal scheduling presented above is easy to implement. As depicted in Fig. 4, link i would probe the channel by sending an RTS packet with probability p_i. The ith receiver would send a CTS packet only if the rate $R_{i,n}$ is good. If the transmitter hears no CTS packets from either its own receiver or other receivers, which indicates that all links are suffering bad channel conditions due to possible deep fading or strong co-channel interference, it would probe the channel again with probability p_i. If after a probing, its channel condition is good, the receiver would send a CTS packet to the transmitter. The transmitters that receive CTS packets from their own receivers (links 1 and 3 in Fig. 4) would transmit data packets. Those transmitters that do not hear from their own receivers, but receive CTS packets from other receivers (link 2 and M) would keep silent until the data transmission is finished. In this study, we assume that the RTS-CTS packets can always be successfully decoded since these control packets are transmitted at a rate that is sufficiently low to ensure correct reception.

B. Optimal Stopping Rule for PHY-aware Distributed Scheduling

For a single-hop ad hoc network with M links, the total reward at time n is $\sum_{j\in K_n} R_{j,n}T$, and the average network throughput is given by

$$x = \frac{E\left[\sum_{j\in K_N} R_{j,N}T\right]}{E[T_N]}. \tag{9}$$

It then follows that for link i, the problem of maximizing the average network throughput can be cast as a maximal-rate-of-return problem, in which a key step is to characterize the optimal stopping rule N_i^* as

$$N_i^* = \arg\max_{N_i\in Q} \frac{E\left[\sum_{j\in K_{N_i}} R_{j,N_i}T\right]}{E[T_{N_i}]}. \tag{10}$$

Clearly, every link has its individual optimal stopping rule, a characteristic that distinguishes it sharply from the case with the collision model [20]. Furthermore, in a single-hop network, all links stop channel probing if at least one link starts to transmit data. Accordingly, the optimal stopping rule of the network for PHY-aware distributed scheduling would be the earliest one among all links, i.e.,

$$N^* = \min_i\{N_i^*,\ i = 1, 2, \cdots, M\}. \tag{11}$$

Next, we use optimal stopping theory to solve the problem in (10). Observe that the reward $\sum_{j\in K_{N_i}} R_{j,N_i}$ is a random variable and is not known at link i. However, it can be shown that replacing the random reward with its conditional expectation $E\left[\sum_{j\in K_{N_i}} R_{j,N_i}|R_{i,n}\right]$ would give the same average return [8]. We then have the following proposition.

Proposition 3.1: The optimal stopping rule N_i^* for PHY-aware distributed scheduling exists, and is given by

$$N_i^* = \min\left\{n \geq 1 : R_{i,n} \geq x^* - E\left[Y_{i,n}|R_{i,n}\right]\right\}. \quad (12)$$

where $Y_{i,n} = \sum_{j\in K_n\setminus i} R_{j,n}$, and x^* is the unique solution to

$$E\left(\sum_{i\in K_n} R_{i,n} - x\right)^+ = \frac{x\tau}{T}. \quad (13)$$

The proof is relegated to Appendix A.

Proposition 3.1 reveals that the optimal stopping rule for PHY-aware distributed scheduling under the physical interference model is a threshold-based policy. It is worth noting that different from the case in the collision model, where the optimal thresholds are the same across different links, *under the physical interference model, different links have different rate thresholds in general*. Furthermore, define

$$g_i(R_{i,n}) \triangleq R_{i,n} + E[Y_{i,n}|R_{i,n}].$$

Then (12) can be rewritten as

$$N_i^* = \min\left\{n \geq 1 : g_i(R_{i,n}) \geq x^*\right\}. \quad (14)$$

Accordingly, instead of making the stopping decision based on its instantaneous data rate, each link would compute $g_i(R_{i,n})$ based on local information. If $g_i(R_{i,n})$ is greater than x^*, it would transmit the data with rate $R_{i,n}$; otherwise, it skips this transmission opportunity. In summary, all links have the same threshold x^*, but use different rate functions $\{g_i(\cdot)\}$ to compare with it.

Unfortunately, Proposition 3.1 does not offer a closed-form expression for the optimal rate threshold x^*. In the following, we develop algorithms for computing the optimal rate threshold. We first look at the basic case with fixed contention probability, and focus on optimizing the rate threshold.

C. Optimal Rate Threshold with Fixed Contention Probability

Let $\phi(x)$ denote the corresponding network throughput with threshold x, where link i would transmit if $g_i(R_{i,n}) \geq x$. Then, the problem of finding the optimal stopping rule for PHY-aware distributed scheduling boils down to finding the optimal threshold x^* that can maximize the network throughput, i.e., $x^* = \arg\max_x \phi(x)$.

It follows that $q_{S,K}$ defined in (2) can be rewritten as

$$q_{S,K} = \prod_{i\in K}(1 - F_S^i(x)) \prod_{j\notin K} F_S^j(x), \quad (15)$$

where $F_S^i(x) \triangleq Pr\{g_i(R_i) \leq x|S\}$.

We then have the following network throughput, given the set of contending links S.

Proposition 3.2: Given that the links in S contending for the channel, the network throughput is given by

$$\phi(x|S) = \frac{\sum_K q_{S,K} \sum_{i\in K} E[R_i]}{\delta + 1 - q_{S,0}}, \quad (16)$$

where $q_{S,0}$ is the probability that the set K is empty, with $q_{S,0} = \prod_{i\in S} F_S^i(x)$, and $E[R_i]$ is given by

$$E[R_i] = \frac{1}{1 - F_S^i(x_i)} \sum_{l=1}^{L} R^l p_{S,l}^i \mathbf{I}(g_i(R^l) \geq x), \quad (17)$$

where $R^0 = 0$, $p_{S,l}^i \triangleq Pr\{R_i = R^l|S\}$.

Finally, combining (1) and (16), and removing the condition on number of contending links S, we obtain the network throughput as

$$\phi(x) = \frac{\sum_S Pr(S) \sum_K q_{S,K} \sum_{i\in K} E[R_i]}{\delta + \sum_S Pr(S)(1 - q_{S,0})}. \quad (18)$$

Recall that the transmission rates are (quantized) discrete values as $R_i \in \{R^l, l = 0, 1, \cdots, L\}$. It follows that $g_i(R_i)$ is in the set $\{g_i(R^l), l = 0, 1, \cdots, L\}$. As a result, the maximum throughput is unique, but the optimal rate thresholds may not be unique in general, since changing the threshold in between two adjacent quantization levels would not affect its optimality, and the new threshold policy achieves the same throughput. In other words, the thresholds in between two adjacent quantization levels are effectively the same. It follows that the possible optimal threshold are in the discrete set $\{g_i(R^l)|i = 1, 2, \cdots, M,\ l = 0, 1, \cdots, L\}$ with at most $M(L+1)$ elements. Therefore, exhaustive search can be used to find the optimal rate threshold in $O(ML)$ time.

D. Joint Optimal Rate Threshold and Optimal Contention Probability

In the previous study, the contention probability of each link was fixed. It is worth noting that the network throughput heavily depends on the contention probability of each link. Specifically, a smaller contention probability would decrease the number of contending links, and the corresponding throughput would be smaller. On the other hand, a larger contention probability would increase the number of contending links, which may incur a stronger co-channel interference. Therefore, one would expect that there exist optimal contending probabilities that maximize the network throughput. Note that different links may have different contention probabilities, denoted as $\mathbf{p} = [p_1, p_2, \cdots, p_M]$. Then, the PHY-aware distributed scheduling problem can be reformulated as finding the optimal pair $(\mathbf{p}, x)$ that maximizes the network throughput as

$$\textbf{P1} \quad \max_{\mathbf{p},x} \quad \phi(\mathbf{p}, x),$$
$$\text{subject to} \quad 0 \leq \mathbf{p} \leq 1,$$
$$x \in \{g_i(R^l)|i = 1, \cdots, M,\ l = 0, \cdots, L\}.$$

In general, it is challenging to characterize the optimal $\mathbf{p}$ and x simultaneously. In the following, we propose a two-stage algorithm to characterize the globally optimal pair $(\mathbf{p}^*, x^*)$.

Stage I: On a faster time scale, we fix the rate threshold as $\hat{x}$, and compute its corresponding optimal contention probability $\mathbf{p}^*(\hat{x})$ as

$$\mathbf{p}^*(\hat{x}) = \arg\max_{0\leq\mathbf{p}\leq 1} \phi(\mathbf{p}, \hat{x}). \quad (19)$$

Stage II: As elaborated above, the optimal rate threshold is in the discrete set $x \in \{g_i(R^l)|i = 1, 2, \cdots, M,\ l =$

$0, 1, \cdots, L\}$, and the number of elements in the set is at most $M(L+1)$. On a slower time scale, we exhaustively search the set $\{(\mathbf{p}^*(\hat{x}), \hat{x}) \mid \hat{x} = g_i(R^l), i = 1, 2, \cdots, M,\ l = 0, 1, \cdots, L\}$ to obtain the global optimal pair of rate threshold and contention probability, $(\mathbf{p}^*, x^*)$, i.e.,

$$\begin{aligned} x^* &= \arg\max_{\hat{x}} \phi(\mathbf{p}^*(\hat{x}), \hat{x}), \\ \mathbf{p}^* &= \mathbf{p}^*(x^*). \end{aligned} \tag{20}$$

The next key step is to solve the optimization problem (19), which is equivalent to the following optimization problem as

$$\begin{aligned} \mathbf{P2} \qquad & \max_{\mathbf{p}} \quad \phi(\mathbf{p}, \hat{x}), \\ & \text{subject to} \quad 0 \leq \mathbf{p} \leq 1. \end{aligned} \tag{21}$$

The above optimization problem is generally non-convex. Direct maximization of $\phi(\mathbf{p}, \hat{x})$ is often prohibitive, since it takes the form of $U(\mathbf{p}, \hat{x})/W(\mathbf{p}, \hat{x})$. Instead, we resort to the technique of fractional maximization to compute $\mathbf{p}^*(\hat{x})$ [3]. To this end, define

$$\begin{aligned} U(\mathbf{p}, \hat{x}) &\triangleq \sum_S Pr(S) c_S \\ W(\mathbf{p}, \hat{x}) &\triangleq \delta + \sum_S Pr(S) d_S, \end{aligned} \tag{22}$$

where c_S and d_S are coefficients independent of $\mathbf{p}$, and defined as

$$\begin{aligned} c_S &\triangleq \sum_K q_{S,K} \sum_{i \in K} E[R_i], \\ d_S &\triangleq (1 - q_{S,0}), \end{aligned} \tag{23}$$

and

$$V(\mathbf{p}, \lambda, \hat{x}) = U(\mathbf{p}, \hat{x}) - \lambda W(\mathbf{p}, \hat{x}), \tag{24}$$

where λ is a real positive value. For convenience, with a slight abuse of notation, we use $V(\mathbf{p}, \lambda)$ instead of $V(\mathbf{p}, \lambda, \hat{x})$ in the following.

For a given λ, let $\mathbf{p}(\lambda)$ denote the contention probability that maximizes $V(\mathbf{p}, \lambda)$ as

$$\mathbf{p}(\lambda) = \arg\max_{\mathbf{p}} V(\mathbf{p}, \lambda). \tag{25}$$

Let $V(\lambda) = V(\mathbf{p}(\lambda), \lambda)$, and λ^* denote the solution to $V(\lambda) = 0$. It can be shown that λ^* is the maximal throughput, and $\mathbf{p}(\lambda^*)$ is the optimal contention probability that maximizes $\phi(\mathbf{p}, \hat{x})$. However, it is generally difficult to find the root of $V(\lambda) = 0$, we then come up with an iterative algorithm to compute λ^*. First, we need the following lemma.

Lemma 3.2: $V(\lambda)$ is decreasing and convex in λ.

The proof follows from Lemma 1 of Chapter 6 in [8] and is omitted here.

Next, we propose an iterative algorithm using Newton's method:

$$\lambda_{t+1} = \lambda_t - \frac{V(\lambda_t)}{V'(\lambda_t)} = \lambda_t + \frac{V(\lambda_t)}{W(\mathbf{p}(\lambda_t))} = \frac{U(\mathbf{p}(\lambda_t))}{W(\mathbf{p}(\lambda_t))} = \phi(\mathbf{p}(\lambda_t)).$$

Given any initial value λ_0, the above iterative algorithm is known to converge quadratically to the optimal throughput λ^* [3]. In summary, we have the following proposition.

Proposition 3.3: Given any positive initial value λ_0, the following iterative algorithm

$$\begin{aligned} \mathbf{p}_t &= \arg\max_{0 \leq \mathbf{p} \leq 1} V(\mathbf{p}, \lambda_t), \qquad (26) \\ \lambda_{t+1} &= \phi(\mathbf{p}_t), \qquad (27) \end{aligned}$$

converges quadratically, and in particular, $\mathbf{p}_t \to \mathbf{p}^*(\hat{x})$, and $\lambda_t \to \lambda^*$, as $t \to \infty$.

Based on Proposition 3.3, the next step is to solve the optimization problem in (26). Rewrite the optimization problem as

$$\begin{aligned} \mathbf{P3} \qquad & \max_{\mathbf{p}} \quad V(\mathbf{p}, \lambda_t), \\ & \text{subject to} \quad 0 \leq \mathbf{p} \leq 1. \end{aligned} \tag{28}$$

Unfortunately, the above optimization problem is generally non-convex. We need to transform the problem to a convex program problem. To this end, rewrite $V(\mathbf{p}, \lambda_t)$ as

$$\begin{aligned} V(\mathbf{p}, \lambda_t) &= U(\mathbf{p}) - \lambda_t W(\mathbf{p}) \\ &\triangleq \sum_S Pr(S) b_S - \delta \lambda_t, \end{aligned} \tag{29}$$

where $b_S = c_S - \lambda_t d_S$ are coefficients irrelevant with $\mathbf{p}$.

Case 1: If $b_S \leq 0$, $V(\mathbf{p}, \lambda_t)$ becomes

$$V(\mathbf{p}, \lambda_t) = \sum_S b_S \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) - \delta \lambda_t, \tag{30}$$

The optimization problem in (28) is equivalent to

$$\begin{aligned} & \max_{\mathbf{p}} \quad \sum_S b_S \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) - \delta \lambda_t, \\ & \text{subject to} \quad 0 \leq \mathbf{p} \leq 1, \end{aligned} \tag{31}$$

It is clear that the maximum is achieved when $p_i = 0, \forall\, i$.

Case 2: If $b_S > 0$ for some S, define

$$b_{max} \triangleq \max\{b_S,\ \forall\ S\} > 0.$$

Let $\emptyset$ denote the empty set with $Pr(\emptyset) = \prod_j (1 - p_j)$. With the fact that $\sum_S Pr(S) + Pr(\emptyset) = 1$, $V(\mathbf{p}, \lambda_t)$ can be rewritten to

$$\begin{aligned} V(\mathbf{p}, \lambda_t) &= \sum_S b_S Pr(S) - \delta \lambda_t + b_{max} Pr(\emptyset) - b_{max} Pr(\emptyset) \\ &= \sum_S b_{max} Pr(S) + \sum_S (b_S - b_{max}) Pr(S) \\ &\quad + b_{max} Pr(\emptyset) - b_{max} Pr(\emptyset) - \delta \lambda_t \\ &= b_{max} - \left[\sum_S \hat{b}_S Pr(S) + b_{max} Pr(\emptyset) \right] - \delta \lambda_t, \end{aligned}$$

with $\hat{b}_S \triangleq b_{max} - b_s \geq 0, \forall\, S$. The optimization problem in (28) is then equivalent to

$$\begin{aligned} & \min_{\mathbf{p}} \quad \sum_S \hat{b}_S \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) + b_{max} \prod_j (1 - p_j), \\ & \text{subject to} \quad 0 \leq \mathbf{p} \leq 1, \end{aligned} \tag{32}$$

which can be transformed into a convex program.

Finally, the algorithm for computing the optimal $(\mathbf{p}^*, x^*)$ is summarized in Algorithm I, as shown in Fig. 5.

Remarks: It is worth noting that computing the optimal rate threshold requires global information. However, the optimal rate threshold depends on statistical information only, and can be computed before network deployment.

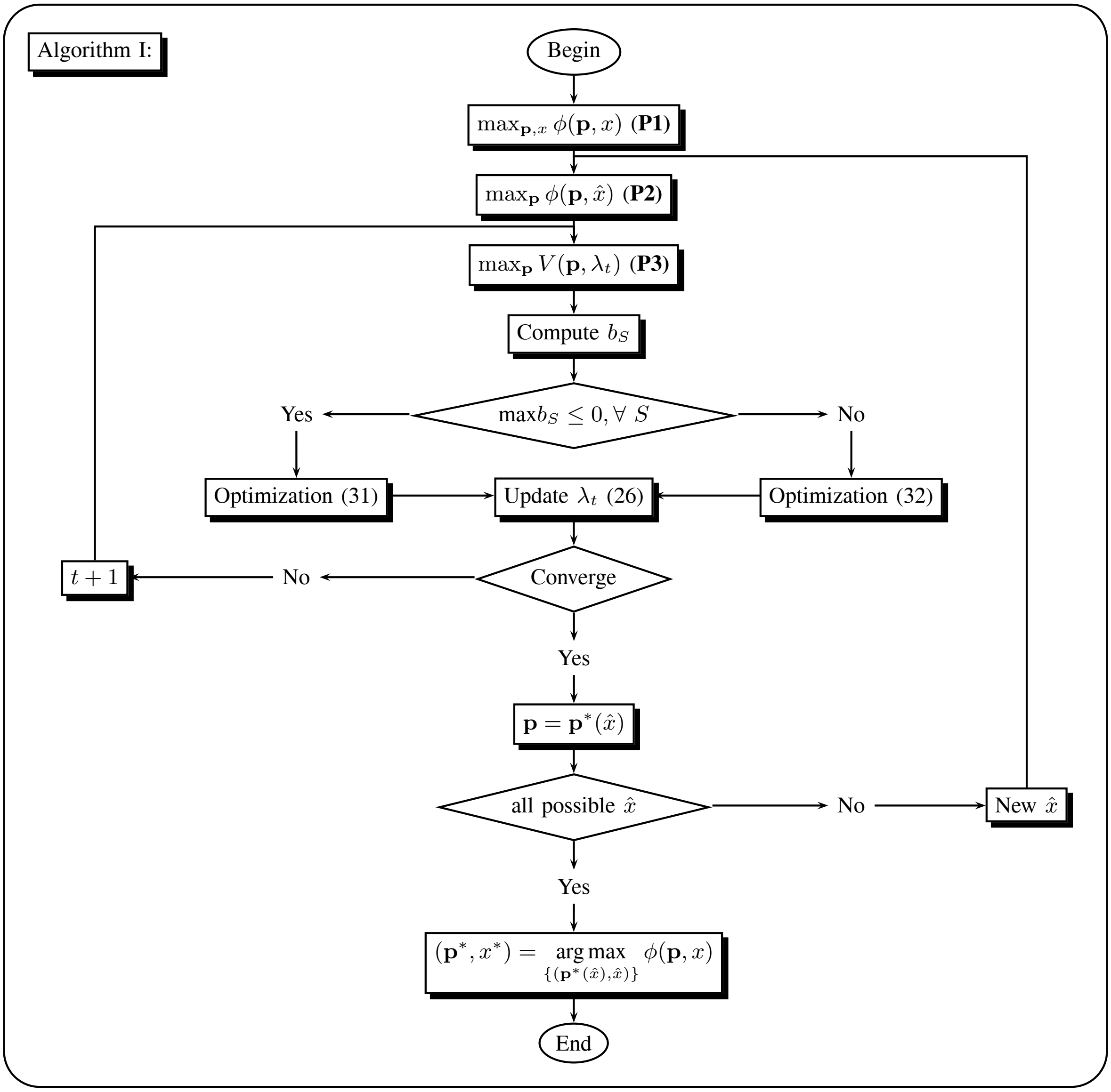


Fig. 5. The flowchart of Algorithm I for computing the optimal pair $(\mathbf{p}^*, x^*)$.

IV. PHY-aware Distributed Scheduling: The Symmetric Channel Model

In this section, we study PHY-aware distributed scheduling with symmetric channels. The channel is said to be symmetric if $q_{K,S}$ depends only on the number of links in S and K. In other words, the channels are statistically homogeneous. It is clear that the optimal threshold and the optimal contention probability are the same across the links for symmetric channel.

Accordingly, for the symmetric channel model, $q_{S,K}$ changes to the probabilistic reception matrix in [10]

$$\pi = \begin{pmatrix} \pi_{1,0} & \pi_{1,1} & & \\ \pi_{2,0} & \pi_{2,1} & \pi_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{33}$$

where $\pi_{m,k}$ is the conditional probability that k links are "good" given that m links are contending for channel access. Let π_0 denote the reception matrix for the conventional collision model and π_1 denote the reception matrix for perfect packet separation as

$$\pi_0 = \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & 0 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \pi_1 = \begin{pmatrix} 0 & 1 & & \\ 0 & 0 & 1 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

With threshold x, the probability that k links transmit given that m links contend for the channel access, $\pi_{m,k}$, is given by

$$\pi_{m,k} = \binom{m}{k} F_m(x)^{m-k}(1 - F_m(x))^k, \tag{34}$$

where $F_m(x)$ is defined as

$$F_m(x) \triangleq \Pr\{g(R_i) < x | m \text{ links probe the channel}\} \tag{35}$$

The average network throughput with symmetric channels is given by

$$\phi(x) = \frac{\sum_{m=1}^{M} \frac{q_m}{1-F_m(x)} \sum_{k=1}^{m} \pi_{m,k} k E[R]}{\delta + \sum_{m=1}^{M} q_m(1-\pi_{m,0})}, \tag{36}$$

where q_m is the probability that exact m links probe the channel and is given by

$$q_m = \binom{M}{m} p^m (1-p)^{(M-m)}, \tag{37}$$

and $E[R]$ is given by

$$E[R] = \sum_{l=1}^{L} R^l p_{m,l} \mathbf{I}(g(R^l) \geq x), \tag{38}$$

where $p_{m,l} \triangleq \Pr\{R = R^l | m \text{ links probe}\}$.

It is clear that the possible optimal threshold is in the discrete set $\{g(R^l),\ l = 0, 1, \cdots, L\}$. The optimization problem **P1** then boils down to

$$\begin{aligned} \mathbf{P4} \quad & \max_{p,x} \quad \phi(p, x), \\ & \text{subject to} \quad 0 \leq p \leq 1, \\ & \qquad x \in \{g(R^l),\ l = 0, 1, \cdots, L\}. \end{aligned} \tag{39}$$

As discussed above, we can apply the two-time scale algorithm to compute the optimal (p, x) pair as the following. On a faster time scale, we fix the threshold as $\hat{x}$, and compute its corresponding optimal contention probability $p^*(\hat{x})$ as

$$p^*(\hat{x}) = \arg\max_{0 \leq p \leq 1} \phi(p, \hat{x}). \tag{40}$$

On a slower time scale, we exhaustively search the set $\{(p^*(\hat{x}), \hat{x}) \mid \hat{x} = g(R^l),\ l = 0, 1, \cdots, L\}$ to obtain the optimal (p^*, x^*) in $O(L)$ time, i.e.,

$$\begin{aligned} x^* &= \arg\max_{\hat{x}} \phi(p^*(\hat{x}), \hat{x}), \quad l = 0, 1, \cdots, L \\ p^* &= p^*(x^*). \end{aligned} \tag{41}$$

The next key step is to solve the optimization problem (40), which is equivalent to the following optimization problem

$$\begin{aligned} \mathbf{P5} \quad & \max_{p} \quad \phi(p, \hat{x}), \\ & \text{subject to} \quad 0 \leq p \leq 1. \end{aligned} \tag{42}$$

To convert **P5** into a convex programming problem, rewrite $\phi(p, \hat{x})$ as

$$\phi(p, \hat{x}) = \frac{\sum_{m=1}^{M} q_m C_m}{\delta + \sum_{m=1}^{M} q_m D_m} \tag{43}$$

where

$$\begin{aligned} C_m &\triangleq \frac{1}{1-F_m(x)} \sum_{k=1}^{m} \pi_{m,k} k E[R], \\ D_m &\triangleq 1 - \pi_{m,0}, \end{aligned} \tag{44}$$

with $C_m \geq 0,\ D_m \geq 0,\ \forall\ m$. Define

$$C_{max} \triangleq \max\{C_m,\ m = 1, 2, \cdots, M\}. \tag{45}$$

Then, with the fact that $\sum_{m=0}^{M} q_m = 1$, $\sum_{m=1}^{M} C_m q_m$ boils down to

$$\begin{aligned} \sum_{m=1}^{M} C_m q_m &= \sum_{m=1}^{M} C_m q_m + C_{max} q_0 - C_{max} q_0 \\ &= C_{max} - \sum_{m=0}^{M} \hat{C}_m q_m, \end{aligned} \tag{46}$$

where

$$\hat{C}_m \triangleq \begin{cases} C_{max}, & m = 0, \\ C_{max} - C_m, & m = 1, 2, \cdots, M. \end{cases} \tag{47}$$

Define $\hat{p} = 1 - p$. The optimization problem **P5** is then equivalent to

$$\begin{aligned} \mathbf{P6} \quad & \min_{p,\hat{p}} \quad \frac{\delta + \sum_{m=1}^{M} D_m q_m}{C_{max} - \sum_{m=0}^{M} \hat{C}_m q_m}, \\ & \text{subject to} \quad p + \hat{p} \geq 1, \\ & \qquad 0 < p \leq 1. \end{aligned} \tag{48}$$

It can be shown that the objective function in **P6** is a monotonically increasing function of p and $\hat{p}$. As a consequence, **P6** can be solved by using the *bisection* method [4].

Finally, the algorithm for computing the optimal (p^*, x^*) is summarized in Algorithm II, as shown in Fig. 6.

V. Discussion on Further Enhancement

Thus far in this paper, we investigated PHY-aware distributed scheduling, in which each link transmits data based on a threshold policy, i.e., link i would transmit data after the Nth probing if $g_i(R_{i,N}) > x^*$. In this section, we propose an enhanced scheme to let each link make decision based on its own channel information and partial information from other links. Recall that the SINR of link i after channel probing is given by

$$\begin{aligned} \text{SINR}_i^p &\triangleq \frac{G_{ii} P}{\sum_{j \in S \setminus i} G_{ji} P + N} \\ &= \frac{G_{ii} P}{\sum_{j \in K \setminus i} G_{ji} P + \sum_{j \in S \setminus K} G_{ji} P + N}. \end{aligned} \tag{49}$$

After channel probing, only the links in K would transmit, and all other links would skip the transmission opportunity (e.g., by skipping CTS) and keep silent over a duration of T. As a result, the SINR of link i during transmission, denoted as SINR_i^r, is given by

$$\text{SINR}_i^r = \frac{G_{ii} P}{\sum_{j \in K \setminus i} G_{ji} P + N}, \quad i \in A. \tag{50}$$

It is clear that $\text{SINR}_i^r \geq \text{SINR}_i^p$. This may cause a severe performance loss, since the channel can support more links to transmit simultaneously. More specifically, consider the example in Fig. 3; after the Nth probing, link 1 has a good SINR condition and would transmit, while link 2 has to keep silent. The total rate for this transmission is $R_{1,N}$. An alternative scheme is to let both links transmit with the total rate of $R_{1,N} + R_{2,N}$. Compared to the distributed scheduling, the new enhanced scheme brings an extra reward $R_{2,N}$. Generally, after a channel probing, some links with good SINR conditions will transmit data with total rate $R_n = \sum_{i \in K_n} R_{i,n}$. Those links with "poor" SINR conditions can also participate in the

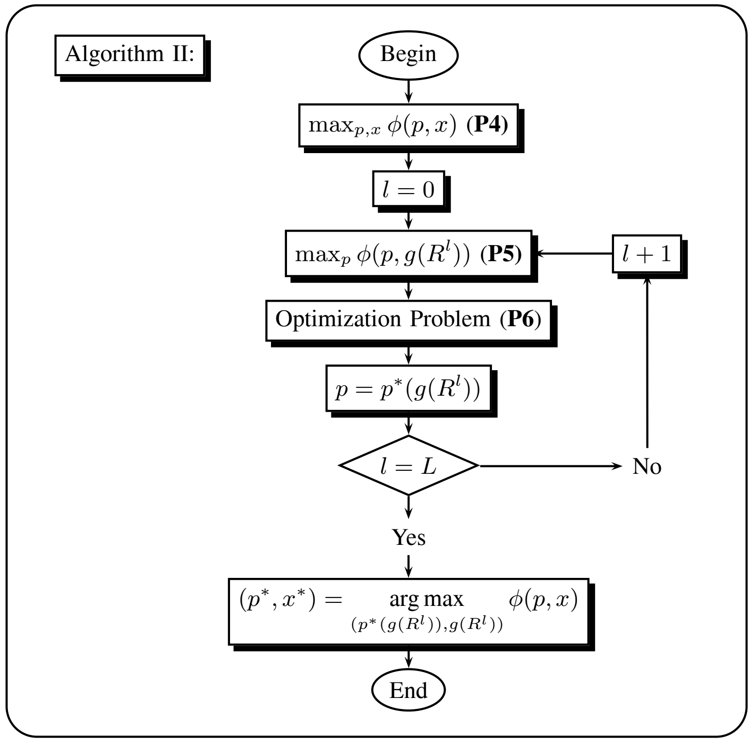


Fig. 6. The flowchart of Algorithm II for computing the optimal (p^*, x^*).

transmission, regardless of their SINR condition. As a result, the total rate is enhanced to $\sum_{i \in S_n} R_{i,n}$, which contributes an extra reward of $\sum_{i \in S_n \setminus K_n} R_{i,n}$.

VI. Numerical Results

In this section, we illustrate, via numerical examples, the performance gain by using the PHY-aware distributed scheduling in a random-access-based ad hoc network under the physical interference model. We note that the optimal policy for PHY-aware distributed scheduling has a threshold structure with positive rate thresholds. For ease of exposition, we study an ad hoc network with Rayleigh fading channels. Let $\rho_{s,i}$ and ρ_n denote the average SNR of the desired signal of link i and the interference signal, respectively. It follows that $\{G_{ii}\}$ and $\{G_{ij}, \forall\, i \neq j\}$ are independent exponentially distributed random variables with parameter $1/\rho_{s,i}$ and $1/\rho_n$, respectively. Unless otherwise specified, we set $M = 5$, $\delta = 0.1$, $\rho_n = 0dB$, and the transmission rates can be 1Mbps, 2Mbps, 5.5Mbps and 11Mbps, with the SINR thresholds given by $\gamma_1 = 0dB$, $\gamma_2 = 5dB$, $\gamma_{5.5} = 10dB$ and $\gamma_{11} = 15dB$.

Fig. 7 depicts the maximal throughput as a function of $\rho_{s,1}$. For the sake of comparison, we also show the network throughput under the conventional scheme, which can be viewed as one with rate threshold zero. Inspection of Fig. 7 confirms that both the distributed scheduling and the enhanced scheme achieve substantial performance gain. Particularly, at $\rho_{s,1} = 30$, the gain is 30% and 17% for the enhanced scheme and PHY-aware distributed scheduling, respectively.

In Table I, for a given rate threshold $x = 2Mbps$, we examine the convergence behavior of the contention probability in the iterative algorithm of (26). It can be seen that the convergence rate is fast, and the solution approaches p^* typically within four iterations. Moreover, the link with the best channel statistics (link 1 in this example) exhibits the largest optimal contention probability.

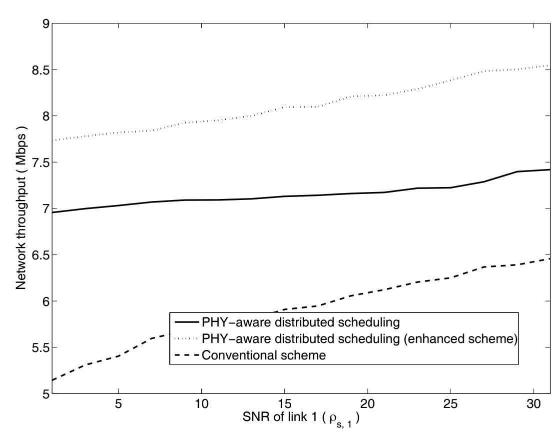


Fig. 7. Maximal throughput as a function of SNR $\rho_{s,1}$, with $\rho_{s,2}$=10dB, $\rho_{s,3}$=10dB, $\rho_{s,4}$=8dB, $\rho_{s,5}$=20dB.

TABLE I
CONVERGENCE BEHAVIOR OF THE CONTENTION PROBABILITY IN THE ITERATIVE ALGORITHM IN (26).

$\rho_{s,i}$	p_0	p_1	p_2	p_3	p_4	p^*
$\rho_{s,1}$=20dB	0.5	0.63	0.68	0.71	0.76	0.76
$\rho_{s,2}$=10dB	0.5	0.41	0.45	0.47	0.47	0.47
$\rho_{s,3}$=8dB	0.7	0.10	0.27	0.34	0.36	0.36

Next, we take a closer look at the network with a symmetric channel, in which the optimal rate threshold and the optimal contention probability are the same across all links. We first

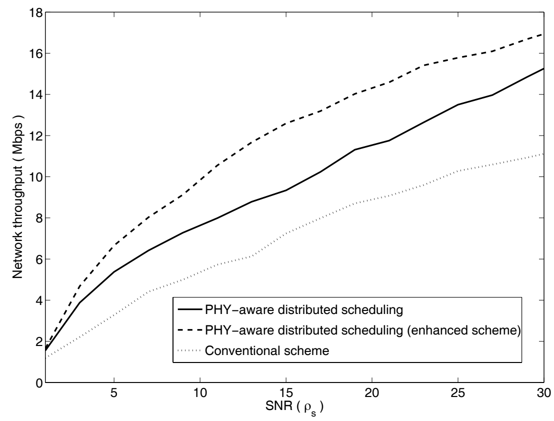


Fig. 8. Maximal throughput as a function of SNR ρ_s

.

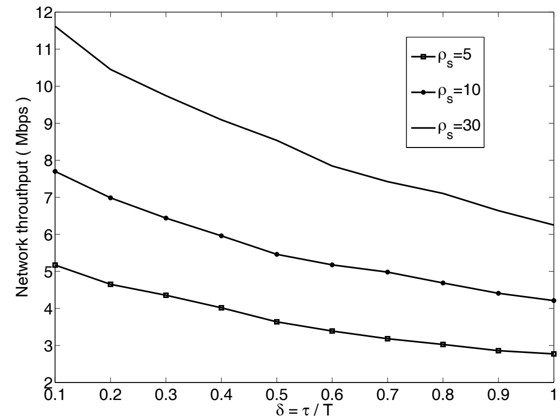


Fig. 9. Network throughput as a function of the probing overhead δ.

TABLE II
CONVERGENCE BEHAVIOR OF THE CONTENTION PROBABILITY IN THE ITERATIVE ALGORITHM IN (26).

ρ_s	p_0	p_1	p_2	p_3	p_4	p_5	p^*
0dB	0.1	0.32	0.58	0.62	0.64	0.65	0.65
10dB	0.5	0.52	0.53	0.53	0.54	0.54	0.54
15 dB	0.7	0.12	0.27	0.32	0.35	0.37	0.37

reexamine the throughput gain and the convergence behavior of the contention probability in Fig. 8 and Table II, respectively. In Fig. 9, we show the impact of the probing overhead $\delta = \tau/T$ on the performance of the PHY-aware distributed scheduling. As expected, the network throughput decreases as δ increases.

In Fig. 10, we illustrate the network throughput as a function of the contention probability. It can be observed that the network throughput is small for both $p = 0$ and $p = 1$, and achieves the maximum somewhere in between. It is well known that under the collision model, the optimal contention probability of each link is $1/M$ [1], which is much smaller than that under the physical interference model ([0.4, 0.6] in this example). Moreover, as the SNR increases, the optimal contention probability also increases. Our intuition is that when SNR is large, links are more robust to cochannel interference, and thus can tolerate more simultaneous transmissions.

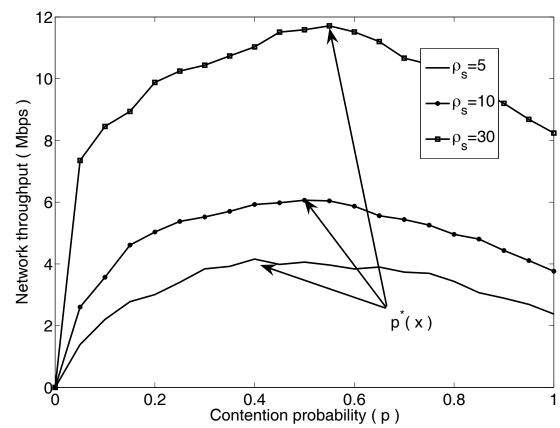


Fig. 10. Network throughput as a function of the contention. probability

VII. CONCLUSIONS

In this paper, we considered a random-access-based ad hoc network model, where links use mini-slots to contend for the channel, and then proceed with data transmission, once they acquire the channel. We focused on developing PHY-aware distributed scheduling for ad hoc communications under the physical interference model. In such a network, PHY-aware distributed scheduling boils down to a process of joint channel probing and distributed scheduling. We then investigated the problem from a network-centric point of view, with the objective of maximizing the overall network throughput. Specifically, we formulated the problem as a maximum-rate-of-return problem by using optimal stopping theory, and demonstrated that the optimal strategy for PHY-aware distributed scheduling has a threshold structure. Accordingly, after a channel probing, a link would proceed with data transmission only if a function of its instantaneous rate is greater than the optimal rate threshold. Observing that the network throughput depends on the contention probability of each link, we generalized the problem to optimize the rate threshold and contention probability in a joint manner, and devised a two-stage algorithm to compute them iteratively by using fractional optimization and geometric programming. We paid special attention to homogeneous networks where all links have the same channel statistics and exploited the bisection method for computing the optimal threshold-probability pair. Numerical results showed that significant throughout gain can be achieved through PHY-aware distributed scheduling.

It is of great interest to generalize this study to multi-hop ad hoc networks. We are currently pursuing PHY-aware distributed scheduling along these avenues.

APPENDICES

A. PROOF OF PROPOSITION 3.1

In what follows, we prove Proposition 3.1 using optimal stopping theory [8]. For a given $x > 0$, define the reward function as

$$Z_n(x) \triangleq \sum_{i \in K_n} R_{i,n}T - xT_n. \quad (51)$$

It follows that PHY-aware distributed scheduling boils down to maximizing the rate of return given by

$$x = \frac{E\left[\sum_{i \in K_N} R_{i,N}T\right]}{E[T_N]}. \quad (52)$$

To this end, a key step is then to find an optimal stopping rule $N^*(x)$ for stopping the channel probing and proceeding with data transmission such that

$$\begin{aligned} V^*(x) &= E\left[\sum_{i \in K_{N^*(x)}} R_{i,N^*(x)}T - xT_{N^*(x)}\right] \\ &= \sup_{N \in Q} E\left[\sum_{i \in K_N} R_{i,N}T - xT_N\right]. \quad (53) \end{aligned}$$

In order to establish the existence of optimal stopping rule for channel probing, we refer to Theorem 1 in [[8], Chapter 3]. It follows that $\limsup_{n \to \infty} Z_n \longrightarrow -\infty$, and that

$$\begin{aligned} E[\sup_n Z_n] &= E\left[\sup_n \sum_{i \in K_n} R_{i,n}T - xT - xn\tau\right] \\ &\leq E\left[\sup_n R_{i,n}TM - xT - xn\tau\right] \\ &< \infty, \quad (54) \end{aligned}$$

indicating that $N^*(x)$ exists.

In PHY-aware distributed scheduling, each link makes the decision independently based on local information only. In other words, the i-th link makes decision based on its own rate $R_{i,n}$, *without the knowledge of the instantaneous rate of other links*. Specifically, let $Y_{i,N} = \sum_{j \in K_N \setminus i} R_{j,N}T$, and rewrite $V^*(\alpha)$ as

$$V^*(\alpha) = \sup_{N \in Q} E[R_{i,N}T + Y_{i,N}T - \alpha T_N]. \quad (55)$$

It is clear that $Y_{i,n}$ is a random variable and is not known at link i. However, it can be shown that by replacing the random "reward" $Y_{i,n}$ with its conditional expectation, the optimal stopping rule remains the same [8]. In other words, $Y_{i,n}$ in (55) can be replaced by the conditional expectation of $Y_{i,n}$ as $E[Y_{i,n}|R_{i,n}]$.

Based on optimal stopping theory [8], for link i, the optimal stopping algorithm $N_i^*(x)$ is given by

$$N_i^*(x) = \min\left\{n \geq 1 : R_{i,n}T \geq V^*(x) + xT - E[Y_{i,n}|R_{i,n}]T\right\},$$

where $V^*(x)$ satisfies the following *optimality equation*:

$$E\left[\max\left(\sum_{i \in K_n} R_{i,n}T - xT - x\tau, V^*(x) - x\tau\right)\right] = V^*(x). \quad (56)$$

Note that $V^*(x^*) = 0$, and (56) becomes

$$E\left[\max\left(\sum_{i \in K_n} R_{i,n}T - x^*T, 0\right)\right] = x^*\tau.$$

The optimal stopping rule for channel probing is simplified to

$$N_i^* = \min\left\{n \geq 1 : R_{i,n} \geq x^* - E[Y_{i,n}|R_{i,n}]\right\}, \quad (57)$$

where x^* is the solution to

$$E\left[\sum_{i \in K_n} R_{i,n} - x\right]^+ = \frac{x\tau}{T}. \quad (58)$$

Recall that the probing stops when at least one of the links decide to proceed with data transmission. As a result, $N^* = \min\{N_i^*,\ i = 1, 2, \cdots, M\}$.

Next we show that (58) has a unique solution. Since $0 \leq x < \infty$, the left hand side of (58) strictly decreases from $E[R]$ to 0 as x increases from 0 to ∞, while the right hand side strictly increases from 0 to ∞. Hence, (58) has a unique finite solution, thereby concluding the proof.

References

[1] N. Abramson, "The ALOHA system-another alternative for computer communications," in *Proc. Fall Joint Computer Conf. AFIPS*, vol. 44, pp. 281-285, Apr. 1970.

[2] S. Adireddy and L. Tong, "Exploiting decentralized channel state information for random access," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 537-561, Feb. 2005.

[3] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA: Athena Scientific, 1995.

[4] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 1992.

[5] G. Brar, D. M. Blough, and P. Santi, "Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks," in *Proc. MOBICOM06*, pp. 2-13, 2006.

[6] Y. S. Chow, H. Robbins, and D. Siegmund, *Great Expectations: Theory of Optimal Stopping*. Houghton Mifflin, 1971.

[7] T. ElBatt, "Towards scheduling MIMO links in interference-limited wireless ad-hoc networks," in *Proc. IEEE MILCOM07*, 2007.

[8] T. Ferguson, *Optimal Stopping and Applications* [Online]. Available: http://www.math.ucla.edu/.

[9] L. Fu, S. C. Liew, and J. Huang, "Joint power control and link scheduling in wireless networks for throughput optimization," in *Proc. IEEE ICC*, 2008.

[10] S. Ghez, S. Verdu, and S. Schwartz, "Stability properties of slotted ALOHA with multipacket reception," *IEEE Trans. Auto. Control*, vol. 33, pp. 640-649, 1988.

[11] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer, "Complexity in geometric SINR," in *Proc. MOBIHOC07*, pp. 100-109, 2007.

[12] Z. Ji, Y. Yang, J. Zhou, M. Takai, and R. Bagrodia, "Exploiting medium access diversity in rate adaptive wireless LANs," in *Proc. MOBICOM04*, 2004.

[13] X. Liu, E. K. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Computer Networks*, vol. 41, no. 4, pp. 451-474, Mar. 2003.

[14] J. Luo and A. Ephremides, "On the throughput, capacity, and stability regions of random multiple access," *IEEE Trans. Inform. Theory*, vol. 52, no. 6, pp. 259-2607, June 2006.

[15] M. H. Ngo and V. Krishnamurthy, "Game theoretic cross-layer transmission policies in multipacket reception wireless networks," *IEEE Trans. Signal Processing*, vol. 55, no. 5, pp. 1911-1926, 2007.

[16] G. Nguyen, A. Ephremides, and J. Wieselthier, "On capture in random-access systems," in *Proc. IEEE ISIT06*, pp. 2072-2076, July 2006.

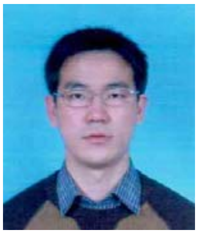
[17] G. Nguyen, J. Wieselthier, and A. Ephremides, "Accurate capture models and their impact on random access in multiple-destination networks," in *Proc. IEEE MILCOM06*, pp. 1-7, 2006.

[18] X. Qin and R. Berry, "Exploiting multiuser dieversity for medium access control in wireless networks," in *Proc. IEEE INFOCOM03*, 2003.

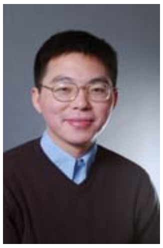
[19] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, pp. 1277-1294, 2002.

[20] D. Zheng, W. Ge, and J. Zhang, "Distributed opportunistic scheduling for ad-hoc communications: An optimal stopping approach," in *Proc. MOBIHOC07*, 2007.

[21] C. Zhu and M. S. Corson, "A distributed channel probing scheme for wireless networks," in *Proc. IEEE INFOCOM01*, vol. 1, pp. 403-411, 2001.



Weiyan Ge received his B.S. in the Department of Electrical Engineering from Peking University (Beijing, China) in 2001, and his M.Phil degree in the Department of Information Engineering from the Chinese University of Hong Kong (Hong Kong) in 2003, and his Ph.D. degree in the Department of Electrical Engineering from Arizona State University (Arizona, USA) in 2008. Currently, he is a Senior Engineer in Qualcomm Inc. San Diego, USA. His research His research interests are in optimization and control of wireless networks.



Junshan Zhang received his Ph.D. degree from the School of ECE at Purdue University in 2000. He joined the EE Department at Arizona State University in August 2000, where he has been Associate Professor since Summer 2005. His research interests include optimization and control of wireless networks, information theory, and stochastic modeling and analysis. His current research focuses on fundamental problems in wireless ad-hoc/sensor networks, including cross-layer optimization and design, resource management, network information theory, and stochastic analysis.

Prof. Zhang is a recipient of the ONR Young Investigator Award in 2005 and the NSF CAREER award in 2003. He has also received the Outstanding Research Award from the IEEE Phoenix Section in 2003. He has served as TPC co-chair for WICON 2008 and IPCCC'06, TPC vice chair for ICCCN'06, and a member of the technical program committees of INFOCOM, SECON, GLOBECOM, ICC, MOBIHOC, BROADNETS, and SPIE ITCOM. He was the general chair for IEEE Communication Theory Workshop 2007. He is an Associate Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and an editor for the COMPUTER NETWORK journal. He co-authored a paper that won IEEE ICC'08 best paper award.

Jeffrey E. Wieselthier was born in Brooklyn, NY, in 1949. He received the S.B. degree from the Massachusetts Institute of Technology, Cambridge, in 1969, the M.S. degree from the Johns Hopkins University, Baltimore, MD, in 1971, and the Ph.D. degree from the University of Maryland, College Park, in 1979, all in electrical engineering. He was employed at the Naval Surface Warfare Center, White Oak, Silver Spring, MD, from 1969 to 1979. From 1979 to 2007 he was with the Information Technology Division of the Naval Research Laboratory, Washington, DC, where he was a Senior Researcher and Head of the Wireless Network Theory Section of the Networks and Communication Systems Branch. Additionally, he was a program manager in communications and networking for the Office of Naval Research. He is currently a self-employed consultant with Wieselthier Research. Dr. Wieselthier was Lead Guest Editor of the 2005 two-part special issue on Wireless Ad Hoc Networks that appeared in the IEEE Journal on Selected Areas in Communications, and was on the Editorial Board of Elsevier's journal Ad Hoc Networks. He was Technical Program Co-Chair of the Third IEEE Symposium on Computers and Communications in Athens, Greece, in 1998 and Treasurer of the 1991 IEEE International Symposium on Information Theory in Budapest, Hungary. He won the IEEE Fred W. Ellersick Award for the best unclassified paper at MILCOM 2000. He has studied a variety of communication networking problems, including multiple access and routing in spread-spectrum networks and the use of neural networks and other approaches for network performance evaluation, optimization and control. His current interests include wireless communication networks, with an emphasis on issues relating to energy-aware, cross-layer, and cooperative operation of ad hoc and sensor networks. Dr. Wieselthier is a member of Eta Kappa Nu and Sigma Xi, and is a Fellow of IEEE.

Xuemin (Sherman) Shen (IEEE M'97-SM'02-F'09) received the B.Sc.(1982) degree from Dalian Maritime University (China) and the M.Sc. (1987) and Ph.D. degrees (1990) from Rutgers University, New Jersey (USA), all in electrical engineering. He is a University Research Chair Professor, Department of Electrical and Computer Engineering, University of Waterloo, Canada. His research focuses on mobility and resource management in interconnected wireless/wired networks, UWB wireless communications networks, wireless network security, wireless body area networks and vehicular ad hoc and sensor networks. He is a co-author of three books, and has published more than 400 papers and book chapters in wireless communications and networks, control and filtering. Dr. Shen is a Distinguished Lecturer of IEEE Communications Society. He serves as the Tutorial Chair for IEEE ICC'08, the Technical Program Committee Chair for IEEE Globecom'07, the General Co-Chair for Chinacom'07 and QShine'06, the Founding Chair for IEEE Communications Society Technical Committee on P2P Communications and Networking. He also serves as a Founding Area Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS; Editor-in-Chief for PEER-TO-PEER NETWORKING AND APPLICATION; Associate Editor for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY; KICS/IEEE JOURNAL OF COMMUNICATIONS AND NETWORKS, COMPUTER NETWORKS; ACM/WIRELESS NETWORKS; and WIRELESS COMMUNICATIONS AND MOBILE COMPUTING (Wiley), etc. He has also served as Guest Editor for IEEE JSAC, IEEE WIRELESS COMMUNICATIONS, IEEE COMMUNICATIONS MAGAZINE, and ACM MOBILE NETWORKS AND APPLICATIONS, etc. Dr. Shen received the Excellent Graduate Supervision Award in 2006, and the Outstanding Performance Award in 2004 and 2008 from the University of Waterloo, the Premier's Research Excellence Award (PREA) in 2003 from the Province of Ontario, Canada, and the Distinguished Performance Award in 2002 and 2007 from the Faculty of Engineering, University of Waterloo. Dr. Shen is a registered Professional Engineer of Ontario, Canada.