

Performance analysis of the cumulative ARQ in IEEE 802.16 networks

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Abstract In this paper, we study the performance of the cumulative Automatic Repeat reQuest (ARQ) in IEEE 802.16 networks. An analytical model is developed to investigate some important performance metrics, such as protocol data unit (PDU) delivery delay, service data unit (SDU) delivery delay, and goodput. A general scheduling scheme and the flexible retransmission of lost PDUs are jointly considered in the analytical model, which provides a more valuable and practical guideline for the system design and performance evaluation. Extensive simulations are conducted to demonstrate the impacts of different operational parameters on the performance metrics and verify the accuracy of the analytical model.

Keywords Cumulative Automatic Repeat reQuest · Scheduling · IEEE 802.16

1 Introduction

Automatic Repeat reQuest (ARQ), a link-layer close-loop error control mechanism, has been identified as an efficient approach to achieve a low packet loss rate in an error-prone

wireless environment. Although ARQ has been extensively studied, new techniques and media access control (MAC) protocol adopted in IEEE 802.16 networks [1, 2] pose fundamental differences in terms of performance analysis of ARQ in IEEE 802.16 networks compared with many previous studies. Firstly, most previous studies on ARQ assumed that the time taken to transmit a packet is a constant, usually referred to as a timeslot. Such an assumption does not hold in IEEE 802.16 networks due to the adoption of adaptive modulation and coding (AMC). With the use of AMC, the link capacity varies with the channel conditions. Therefore, the time taken to transmit a packet varies with channel conditions as well. Secondly, the previous studies commonly assumed that the lost protocol data units (PDUs) are retransmitted in the very next timeslot [3–10]. The analysis of service data unit (SDU) delivery delay for IEEE 802.16 networks discussed [11] is also based on the assumption that the lost PDUs were retransmitted immediately in the next frame. The assumption of immediate retransmission is a strong assumption because the time consumed by data decoding, processing, and feedback is not negligible and the round-trip time should be considered. In addition, the assumption may result in low system capacity due to varying channel conditions. For instance, when the channel condition of the next timeslot is poor, the retransmission could be subject to a high packet loss rate. Thirdly, the previous studies [3–10] do not consider the effect of scheduling scheme. They commonly assumed that the sender continuously transmits PDUs. When a scheduling scheme is taken into account, the retransmission of lost PDUs is flexible and does not just occur within the very next timeslot, which can further improve the system capacity.

In this paper, we study the performance of the cumulative ARQ under the consideration of the general scheduling scheme in IEEE 802.16 networks. An absorbing

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Markov chain and a two-queue model are developed for providing a simple and efficient approach to investigate the important performance metrics, such as PDU delivery delay, SDU delivery delay, and goodput. Furthermore, a general scheduling scheme and flexible retransmission of lost PDUs are jointly considered in the analytical model, which can provide a more valuable and meaningful guidelines for the practical system design and performance evaluation. Extensive simulations are conducted to illustrate the impacts of different operation parameters on performance metrics and to verify the accuracy of the analytical model.

The remainder of the paper is organized as follows. Related works are reviewed in Sect. 2. Section 3 describes the cumulative ARQ in IEEE 802.16 networks by jointly considering the general scheduling scheme. Section 4 presents the developed analytical model. Extensive simulation and numerical results are given in Sect. 5. Finally, we conclude the paper in Sect. 6.

2 Related work

Extensive research efforts have been conducted on the performance analysis of ARQ [3–7]. The study in [3] analyzes the throughput efficiency of different ARQ mechanisms. Vuyst et al. [4] discusses the delay for the stop-and-wait ARQ. The analysis of PDU delay over time-varying wireless channels is given in [5]. The delay for the selective ARQ is discussed in [6]. The study in [7] analyzes the PDU delay with the consideration of the round-trip delay of acknowledgement messages. The aforementioned studies are focused on the delay analysis of PDUs. From the upper-layer's point of view, the knowledge on the SDU delay will be more helpful since an SDU is sent to the upper layer only when all the corresponding PDUs are received and assembled successfully. Otherwise, the SDU is still recognized as “lost” even if part of its PDUs have been successfully received. In [8], SDU delivery delay is analyzed using signal flow graphs without considering the channel model. In [9], the analysis of SDU delivery delay is introduced with the consideration of a Markov channel model. Hou et al. [10] proposes a differentiated ARQ scheme and introduces the performance analysis in terms of SDU loss rate and SDU delay.

Several ARQ schemes have been proposed for different networks. The node cooperative ARQ mechanism is introduced in [12] for ad-hoc networks. By exploiting the channel diversity in wireless scenario, the proposed scheme can efficiently combat the negative impacts of channel fading and improve the system performance. The ARQ initialized cooperative communication protocol is introduced in [13], where the important performance metrics are

analyzed with the consideration of Nakagami-m wireless channel model.

To support high quality video and voice delivery, a cross-layer perceptual ARQ is proposed in [14] to improve the quality of video delivery by jointly considering the application layer information such as the perceptual and temporal importance of each packet. A cross-layer analytical framework is proposed in [15] for voice capacity planning in IEEE 802.11 networks. On the other hand, ARQ is incorporated with the forward error correction (FEC) mechanism to improve system performance. In [16], jointly optimization of FEC and ARQ is discussed for providing video streaming over IEEE 802.11 networks, in which a heuristic algorithm is proposed to select the parameters of FEC with the consideration of ARQ to achieve a high video quality and system efficiency.

In addition, the system performance can be significantly improved by jointly considering the channel conditions in the ARQ mechanism. A stochastic learning automation-based ARQ in [17] uses the learning automaton to predict and track the time-varying wireless channel condition, and makes the decision of transmission or transmission of PDUs based on the history of past observation. The channel-aware ARQ mechanism is proposed in [18] to achieve the energy efficiency and an improved system throughput, in which the channel state information is jointly considered when the system decides to transmit or retransmit PDUs.

3 Cumulative ARQ and scheduling scheme

IEEE 802.16 Standard specifies three ARQ mechanisms: cumulative ARQ, selective ARQ, and hybrid ARQ. This paper focuses on the cumulative ARQ due to its simplicity in implementation. Without loss of generality, the following discussions are for downlink traffic.

At the MAC layer, an SDU is divided into multiple ARQ blocks. By undertaking the process of fragmentation, an SDU is divided into multiple PDUs, and each PDU is composed of sets of blocks. Fragment sequence number (FSN) and block sequence number (BSN) are used to guarantee the proper assembly of the received PDUs at the receiver side. Since PDU is the unit transmitted over the PHY layer, the granularity considered in this paper is PDU, rather than blocks. This consideration is valid and simpler for studying the delay of SDU. The principle of the cumulative ARQ in IEEE 802.16 networks is described as follows.

At the Base Station (BS), the link-layer entity receives SDUs from the upper layer. The received SDUs are classified and buffered into the corresponding queues based on their designated Subscriber Stations (SSs) and their service types. Each SDU is segmented into F PDUs of equal size

prior to delivery over the wireless channel. Time domain is divided into MAC frames with a common duration. Each MAC frame is subdivided into a downlink (DL) sub-frame followed by an uplink (UL) sub-frame. At the beginning of each MAC frame, the BS assigns timeslots for transmitting PDUs designated to each SS according to adopted scheduling scheme. If a queue obtains a chance of transmission, a certain number of PDUs buffered at this queue are transmitted at this DL sub-frame. Otherwise, no transmission is permitted for this queue during this DL sub-frame period.

At the receiver side, the received PDUs are buffered until a complete SDU is successfully received. Each SS sends feedback information back to the BS during each UL sub-frame, indicating whether a PDU is successfully received or not. One of the important feedback information is the FSN, which indicates the sequence number of the last received PDU before the first lost PDU. Based on this feedback information, the BS has the knowledge about the sequence number of the first lost PDU among all PDUs launched in this DL sub-frame, which is $(FSN + 1)$. Then the BS updates the sequence number of its sending window as $(FSN + 1)$. All PDUs launched in this DL sub-frame with a sequence number larger than or equal to $(FSN + 1)$ have to be retransmitted no matter they have been successfully received at the receivers or not. When a scheduling scheme is taken into account, these PDUs are retransmitted when the corresponding queue obtains the transmission opportunity, instead of being retransmitted immediately in the next MAC frame. The flexibility of retransmitting the lost PDUs in IEEE 802.16 networks motivates us to investigate the performance of the cumulative ARQ mechanism by jointly considering a scheduling scheme.

The general scheduling scheme proposed in [19] is considered in the paper, which is briefly described as follows. It is characterized by two parameters: h and L , where h is the number of SSs selected at each MAC frame, while L is the number of PDUs granted to an SS when it is served. Differentiated services among traffic flows in the system can be achieved by using different values of h and L for satisfying their QoS requirements in terms of goodput and delivery delay. At the beginning of each MAC frame, a

number of h SSs with better channel conditions are selected, and each is granted with L PDUs. When h is set as 1, the scheduling scheme becomes the opportunistic scheduling, which can maximize the system throughput at the expense of longer delivery delay of the SSs with poor channel conditions. When h equals to the total number of SSs associated to the BS, it becomes round robin scheduling. The impacts of the parameters, h and L , on the delivery delay and goodput will be considered in the developed analytical model and demonstrated through the extensive simulations.

Figure 1 illustrates the cumulative ARQ mechanism with the aforementioned scheduling scheme for a tagged queue. In the first frame, since the PDU with sequence number 3 has been lost and identified by the receiver, the largest sequence number among all successfully received PDUs is 2. Therefore, in the feedback information, the FSN is 2, and all the PDUs with a sequence number from 3 to L need to be retransmitted. Then, the tagged queue waits for another m frames until it obtains the transmission opportunity again, where m is a random variable in unit of frames and its probability density function is based on channel conditions of all SSs under the consideration. When the tagged queue obtains the transmission opportunity, L PDUs with sequence number from 3 to $L + 2$ are retransmitted/transmitted. Then, the BS updates the FSN accordingly based on the feedback information. Another important parameter in the ARQ mechanism is ARQ window size, which limits the maximum number of unacknowledged ARQ blocks at any given time. However, if the ARQ window size is larger than the value of L , it does not affect the analysis on the delay of SDU, which is what we have considered in the analytical and simulation study.

4 Performance analysis

For simplicity, a queue under consideration is referred to as the *tagged queue*, and the SS to which the PDUs buffered at the tagged queue are destined is referred to as the *tagged SS*. The proposed analytical model is based on the following assumptions:

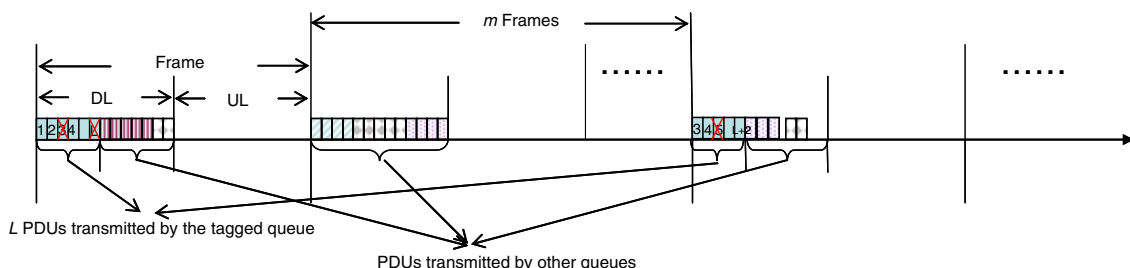


Fig. 1 The illustration of the cumulative ARQ with a scheduling scheme

- (1) in the link layer, each SDU is fragmented to F PDUs with equal size of B bits;
- (2) feedback information of PDUs launched in a DL sub-frame will be sent back to the BS in the following UL sub-frame using the UL-ACK channel, which has been defined in IEEE 802.16e standard [20];
- (3) during each DL sub-frame, h SSs are selected, and the queues destined to these h SSs obtain the transmission opportunities;
- (4) L PDUs are transmitted when the tagged queue obtains the transmission opportunity.
- (5) the lost PDUs will be retransmitted until they are successfully received. Therefore, we can evaluate the worst case of the SDU delivery delay without the constraint on the maximum lifetime or retransmission number.
- (6) when an SS is scheduled, it has PDUs waiting for transmission.

4.1 Wireless channel model

Wireless channels suffer from deep fading that occurs randomly in the time span with a random duration and depth. Numerous studies have shown that such channels can be described by a Markov model to capture such bursty error nature. In this paper, a finite state Markov Channel (FSMC) model, which has been widely adopted in various related research [21–23], is adopted to model the time-varying wireless channel of each SS. In addition, when the channel model is constructed, the discrete adaptive modulation and coding defined in the IEEE 802.16 standard is taken into account, where the received signal-to-noise ratio (SNR) is divided into several disjoint regions, based on a set of boundaries. According to the received SNR of an SS, the BS selects a proper modulation level and coding scheme for this SS. The boundaries for the $(N + 1)$ -state FSMC are denoted as a row vector $\underline{B} = [b_0, b_1, \dots, b_N, b_{N+1}]$, where $b_0 = 0$ and $b_{N+1} = \infty$. When the received SNR is located within the set $[b_n, b_{n+1})$, the channel state is represented by state n ($n = 0, 1, 2, \dots, N$). In this paper, an 8-states Markov channel model is considered. The values of b_n ($n = 0, 1, 2, \dots, 7$) and the modulation and coding levels corresponding to each channel state are listed in Table 1.

The channel states of the FSMC model are abstracted as shown in Fig. 2. ‘0’ is the state without transmission permitted, which happens when the channel condition is very poor. In this case, the corresponding queue is preferred to keep in silence in order to improve the system performance. For simplicity, states BPSK(1/2), QPSK(1/2), ..., and 64QAM(3/4) are represented by states ‘1’, ‘2’, ..., and ‘7’, respectively.

Table 1 State boundaries and corresponding AMC levels in IEEE 802.16 networks

State ID	Modulation level (coding)	b_n (dB)
0	Silent	0
1	BPSK (1/2)	3
2	QPSK (1/2)	6
3	QPSK (3/4)	8.5
4	16QAM (1/2)	11.5
5	16QAM (3/4)	15
6	64QAM (2/3)	18.5
7	64QAM (3/4)	21

The probability of staying at state ‘ n ’ (denoted as $\pi(n)$) is given by [23]

$$\pi(n) = \frac{\Gamma(m, mb_n/\bar{\gamma}) - \Gamma(m, mb_{n+1}/\bar{\gamma})}{\Gamma(m)} \quad (1)$$

where $\bar{\gamma}$ is the average SNR, m is Nakagami fading parameter, $\Gamma(m)$ is the Gamma function, and $\Gamma(m, \gamma)$ is the complementary incomplete Gamma function. Note that a channel becomes a Rayleigh fading channel when $m = 1$. For a slow fading channel, the state transition matrix for the FSMC can be expressed as follows

$$\underline{P} = \begin{bmatrix} p_{00} & p_{01} & 0 & \cdots & 0 & 0 & 0 \\ p_{10} & p_{11} & p_{12} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & & 0 \\ \vdots & \vdots & & \vdots & \vdots & & 0 \\ 0 & 0 & 0 & \cdots & p_{N-2,N-1} & p_{N-1,N-1} & p_{N-1,N} \\ 0 & 0 & 0 & \cdots & 0 & p_{N,N-1} & p_{N,N} \end{bmatrix} \quad (2)$$

The transition probability from state n to k (denoted as $p_{n,k}$) is given as

$$p_{n,n+1} = \frac{L_{n+1} \cdot T}{\pi(n)} \quad n = 1, 2, \dots, N-1. \quad (3)$$

$$p_{n,n-1} = \frac{L_n \cdot T}{\pi(n)} \quad n = 2, 3, \dots, N \quad (4)$$

$$p_{n,n} = \begin{cases} 1 - p_{n,n+1} - p_{n,n-1} & 1 < n < N \\ 1 - p_{1,2} & n = 1 \\ 1 - p_{N,N-1} & n = N, \end{cases} \quad (5)$$

where T is the time duration of each MAC frame, and L_n is the level crossing rate at b_n corresponding to the state n , which can be estimated by

$$L_n = \sqrt{2\pi \frac{mb_n}{\bar{\gamma}}} \cdot \frac{f_d}{\Gamma(m)} \cdot \left(\frac{mb_n}{\bar{\gamma}}\right)^{m-1} \cdot \exp\left(-\frac{mb_n}{\bar{\gamma}}\right) \quad (6)$$

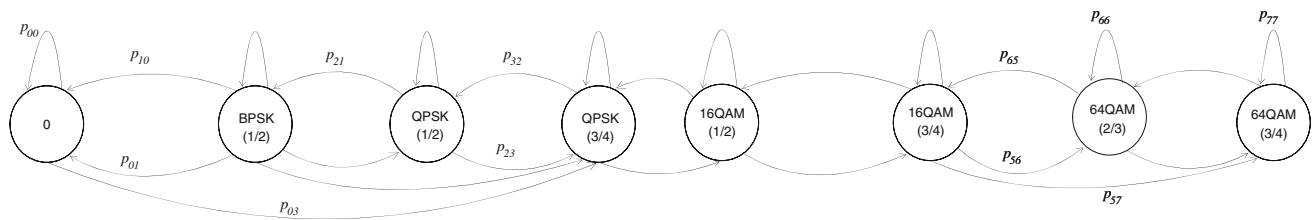


Fig. 2 Finite states Markov channel model

where f_d is the Doppler frequency.

Based on the available modulation and coding schemes in IEEE 802.16 standards, we consider an 8-state Markov channel model in the paper. The finite state Markov channel model and the derivation of the transition probability are also valid in general case and can be extended to any number of states if a finer granularity is required. In other words, the transition probability with any large number of states can be obtained by using Eqs. 3–6 with the expense of a larger state space and longer computing time.

The channel model discussed above is used to derive the inter-service time of SSs, which is required in the performance analysis in terms of goodput, PDU and SDU delivery delay at the following Subsections.

4.2 Analysis of goodput

Goodput achieved at the tagged queue is defined as the average data rate (in the unit of bit/second) successfully launched by the tagged queue. Let μ denote the number of PDUs successfully launched by the tagged queue during a transmission opportunity. The probability mass function of μ is given as

$$pr[\mu = i] = \begin{cases} (1-p)^i p & i = 0, 1, 2, \dots, L-1; \\ (1-p)^L & i = L \end{cases} \quad (7)$$

where p is the error probability of transmitting each PDU, L is the number of PDUs transmitted by the tagged queue during a DL sub-frame. The mean of μ is given by

$$E[\mu] = \sum_{i=0}^{L-1} i (1-p)^i p + L(1-p)^L \quad (8)$$

Let $E[m]$ denote the mean of the inter-service time for the tagged queue, which is defined as the average number of frames between two successive transmission opportunities for the tagged queue. The derivation of $E[m]$ is given in Appendix. Thus, the goodput achieved by the tagged SS is given by

$$G = \frac{E[\mu] \cdot B}{T \cdot (E[m] + 1)} \text{ bps} \quad (9)$$

where T is the time duration of a MAC frame, and B is the size of a PDU in unit of bits.

4.2.1 Analysis of delivery delay of a PDU

We evaluate the delivery delay of a PDU in unit of frame, which is defined as the total number of frames lasting from the first transmission of a PDU to the frame during which this PDU is successfully received. Let N_p and m_i be the number of transmission attempts experienced by the tagged queue to successfully transmit a PDU and the i th inter-service time, respectively. Then, the delivery delay of a PDU can be expressed by

$$D_p = \sum_{i=1}^{N_p-1} (m_i + 1) \quad (10)$$

Hence, the average delivery delay is given by

$$E[D_p] = E \left[\sum_{i=1}^{N_p-1} (m_i + 1) \right] = (E[N_p] - 1)(E[m] + 1) \quad (11)$$

The calculation of $E[N_p]$: we refer to L PDUs that are launched at each DL sub-frame as a *transmission burst*. Based on the principle of cumulative ARQ discussed in Sect. 3, the delivery delay of a PDU is relative to its position at the transmission burst where its first transmission occurs. Note that whether or not a PDU is successfully transmitted depends not only on the successful transmission of itself but also on the successful transmission of all the previous PDUs in the transmission burst where its first transmission occurs. For a PDU whose

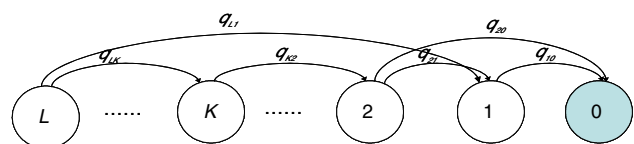


Fig. 3 The transition diagram of a PDU

first transmission occurs at the k th position of a transmission burst, its transmission can be modeled by an absorbing Markov chain as shown in Fig. 3, where the state ' i ' represents that this PDU is at the i th position in a transmission burst, and the state '0' is the absorbing state representing that a PDU is transmitted successfully.

The one-step transition probability matrix is given by

$$Q = \begin{bmatrix} q_{L,L} & q_{L,L-1} & \cdots & q_{L,0} \\ q_{L-1,L} & q_{L-1,L-1} & \cdots & q_{L-1,0} \\ \vdots & \vdots & \ddots & \vdots \\ q_{1,L} & q_{1,L-1} & \cdots & q_{1,0} \\ q_{0,L} & q_{0,L-1} & \cdots & q_{0,0} \end{bmatrix} \quad (12)$$

where q_{ij} is the transition probability from the state i to the state j , which is given by

$$q_{ij} = \begin{cases} 0 & i < j \\ (1-p)^i & j = 0 \\ (1-p)^{(i-j)}p & \text{Otherwise} \end{cases} \quad (13)$$

Hence, the expected number of transmission opportunities for successfully transmitting a PDU is equivalent to the average number of steps experienced by the PDU until to be absorbed, which is given by

$$E[N_P] = \Pi_0 (I - R)^{-1} e \quad (14)$$

where Π_0 is the initial state vector, I is a $L \times L$ identity matrix, R is the matrix derived from the one-step transition probability Q by deleting the row and column corresponding to the absorbing state (0), and e is a $L \times 1$ identity vector.

Let ζ denote the probability that a PDU is located at the i th position in a transmission burst when it is first transmitted. The initial state vector, $\Pi_0 = [\pi_1, \pi_2, \dots, \pi_i, \dots, \pi_L]$, is derived as follows:

$$\begin{aligned} \pi_i &= pr(\zeta = i) = \sum_{j=1}^L pr(\zeta = i, \mu = j) \\ &= \sum_{j=1}^L pr(\mu = j) pr(\zeta = i | \mu = j) \end{aligned} \quad (15)$$

$$pr(\zeta = i | \mu = j) = \begin{cases} 1/j & i = L - j + 1, \dots, L \\ 0 & \text{Otherwise} \end{cases} \quad (16)$$

where μ denotes the number of PDUs successfully transmitted during each time the tagged queue obtains the chance of transmission, and its probability density function has been given by (7).

4.3 Analysis of delivery delay of an SDU/Package

We calculate the deliver delay of an SDU in unit of frame, which is defined as the total number of frames lasting from the first transmission of the first PDU belonging to this

SDU to the frame in which the last PDU of the SDU is successfully received. Let N_S be the number of transmission opportunities for successfully transmitting an SDU, and m_i the i th inter-service time. Then, the delivery delay of an SDU is expressed by

$$D_s = \sum_{i=1}^{N_S-1} (m_i + 1) \quad (17)$$

Hence, the average delivery delay of an SDU is given by

$$E[D_s] = E \left[\sum_{i=1}^{N_S-1} (m_i + 1) \right] = (E[N_S] - 1)(E[m] + 1) \quad (18)$$

where $E[m]$ is the average inter-service time, and $E[N_S]$ is the expected number of transmission chances experienced by the queue to successfully transmit an SDU, which is derived as follows.

The calculation of $E[N_S]$: a two-queue model is developed to evaluate the delivery delay of each SDU at the tagged queue, where two logic queues are referred to as *transmission queue* (tQ) and the *waiting queue* (wQ). The tQ buffers the PDUs to be transmitted at the next transmission opportunity, while wQ buffers other PDUs. We assume that the failed PDUs have a higher priority during the next transmission opportunity. In other words, the PDUs in tQ are composed of all PDUs failed during the previous transmission and the PDUs from the wQ with the number of leftover quota out of L .

We observe the delivery of an arbitrary SDU in the tagged queue, which is referred to as the *tagged SDU*, and all PDUs belonging to the tagged SDU are referred to as the *tagged PDUs*. Let t_1 be the time instant at which the tagged queue wins the transmission opportunity and the first tagged PDU is transmitted in this opportunity. Let the time sequence $\{t_n : n > 1\}$ denote the following successive instants at which the tagged queue obtains the chance of transmission, and $Loc_n \in \{0, 1, \dots, L\}$ and $S_n \in \{0, 1, \dots, F\}$ represent the locations of the first tagged PDUs in the tQ and the total number of the successfully transmitted tagged PDUs observed at the instants $\{t_n : n \geq 1\}$, respectively. The process $\{Loc_n, S_n : n = 1, 2, \dots\}$ forms an absorbing embedded Markov chain on the state space $\{(0, 1, 2, \dots, L) \times (0, 1, 2, \dots, F)\}$, as shown in Fig. 4, which represents the state transition of the tagged PDUs. The state $(0, F)$ is the absorbing state, representing that all the tagged PDUs are transmitted successfully. When the system reaches the absorbing state $(0, F)$, it means that the tagged SDU is transmitted successfully. The one-step transition probability matrix of this Markov chain is given by

$$\Psi = [\varphi_{ij,j'}] \quad i, i' \in \{0, 1, 2, \dots, L\}; j, j' \in \{0, 1, 2, \dots, F\} \quad (19)$$


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for  $i=1,2,\dots,L$ 
  for  $j=1,2,\dots,L$ 
    for  $m=1,2,\dots,L$ 
      for  $n=1,2,\dots,L$ 
        if (initial state  $(m,n)$  occurs with the happening of the transition
            from the state  $i$  to the state  $j$ )
           $K=\text{the number of times the initial state } (m,n) \text{ occurs;}$ 
           $pr(m,n)=pr(m,n)+K\cdot h_{ij}\cdot q_{ij};$ 
        end if
      end for
    end for
  end for
end for
 $S=\text{summation of } pr(m,n) \text{ for all initial states.}$ 
for  $m=1,2,\dots,F$ 
  for  $n=1,2,\dots,F$ 
     $\pi_0(m,n)=pr(m,n)/S;$ 
  end for
end for

```

Fig. 6 The pseudo-code for deriving initial state probability

transition from the state ‘1’ to the state ‘ L ’, which is due to the successful transmission of $(L - 1)$ PDUs at the previous transmission. In other words, the initial state $(2, 0)$, $(F + 2, 0)$, ... $((\lfloor (L - 1)/F \rfloor \cdot F + 2), 0)$ occurs one time simultaneously. Since the transition from state i to state j occurs with the probability $h_{ij}q_{ij}$, the probability by which the corresponding initial states occur can be obtained accordingly. Therefore, the initial state vector Θ_0 can be derived based on the steady-state probability vector H and the one-step transition probability matrix Q . The pseudo-code for the process of deriving the initial state probability vector, Θ_0 , is given in Fig. 6.

When the obtained initial state probability vector Θ_0 is obtained, the average number of steps for an SDU to absorption is derived from (21), and SDU delivery delay is derived from (18).

5 Simulation results

Extensive simulations are conducted to demonstrate the impact of different parameters (i.e., h , L , and p) on the performance metrics in terms of the delivery delay and goodput. Rayleigh flat fading channel is adopted in the simulation. The average SNRs of SSs are given in Table 2. The transmission opportunity of an SS is granted depend on its instantaneous SNR as well as the values of parameters h and L . The other simulation parameters are listed in

Table 2 Average SNR of SSs

Index of SSs	1	2	3	4	5
Average SNR (dB)	10	14	18	20	22

Table 3 Simulation parameters

Frame duration	DL/UL sub-frame duration	Size of a PDU
2 ms	1 ms/1 ms	1056 bits
No. of SSs	OFDM symbol duration	Bandwidth
5	11.46 μ s	20 MHz
F	Doppler frequency	p
5	15 Hz	0.01

Table 3. We repeat the simulation 50 times with different random seeds and calculate the average value. Meanwhile, the confident intervals with 95% confidence level are given to indicate the reliability of the simulation results. In addition, we compare the simulation results (denoted as Sim) with the numerical ones (denoted as Ana) to validate the developed analytical model.

Figures 7 and 8 show the PDU and SDU delivery delays versus L for different p values, respectively. It can be seen that the SDU delivery delay decreases with the increase of L . With a larger L , more PDUs are transmitted when the tagged queue obtains a transmission opportunity, which leads to more resources allocated to the tagged queue and, as a result, a smaller average SDU delay is achieved. On the other hand, with a larger L , the probability of successfully receiving a specific PDU in a MAC frame reduces since more PDUs have to be successfully transmitted before the PDU is successfully launched. Figure 9 shows the goodput of the tagged queue versus L under different PDU loss probability p . With a larger L , more resources are allocated to the tagged queue. Thus, the achieved goodput gets larger. From Figs. 7, 8, and 9, it can be seen that the simulation and analysis results match very well, which justifies the accuracy of the analytical model.

Figures 10, 11, and 12 illustrate the impacts of h on the performance metrics in terms of PDU delivery delay, SDU delivery delay and goodput. From Figs. 10 and 11, it is

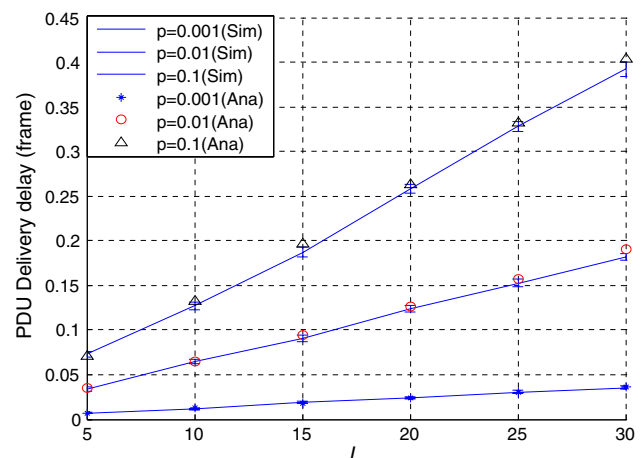


Fig. 7 PDU delivery delay versus L with different p

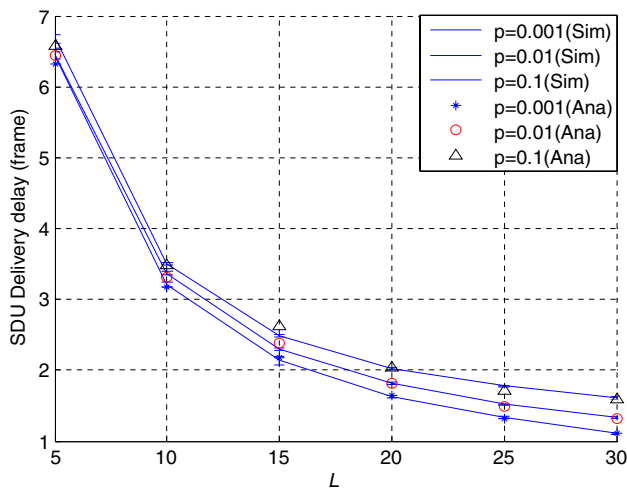


Fig. 8 SDU delivery delay versus L with different p

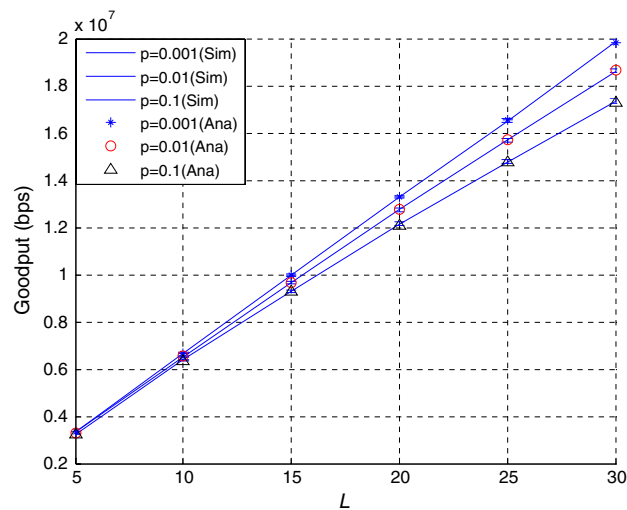


Fig. 9 Goodput versus L for different p

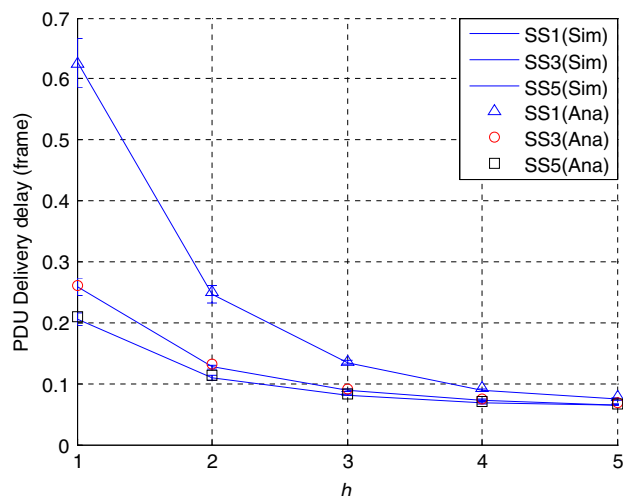


Fig. 10 PDU delivery delay versus h

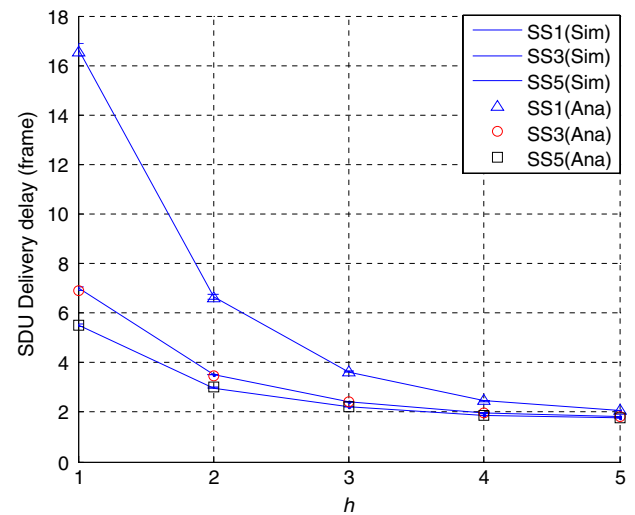


Fig. 11 SDU delivery delay versus h

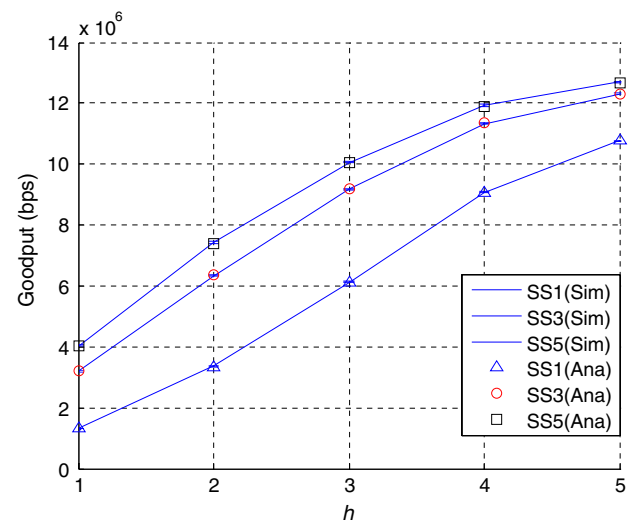


Fig. 12 Goodput versus h

observed that both the PDU delivery delay and SDU delivery delay decrease with the increase of h . A larger h leads to a short inter-service time. In other words, SSs are visited more frequently and have more chances of transmission, which leads to a shorter delivery delay for both PDU and SDU. The opportunistic scheduling is the specific case when h is 1. In this case, SS1, with the worse channel condition, is starved for a long time. Its delivery delay is much larger than that of other SSs. Figure 12 shows the relationship between the obtained goodput and h for different SSs. With the increase of h , inter-service time of each SS decreases. Each SS obtains more chances of transmission, leading to a higher throughput.

Figures 13, 14, and 15 illustrate the impacts of the parameter p on the performance metrics in terms of PDU delivery delay, SDU delay and goodput. From Figs. 13 and 14, it can be seen

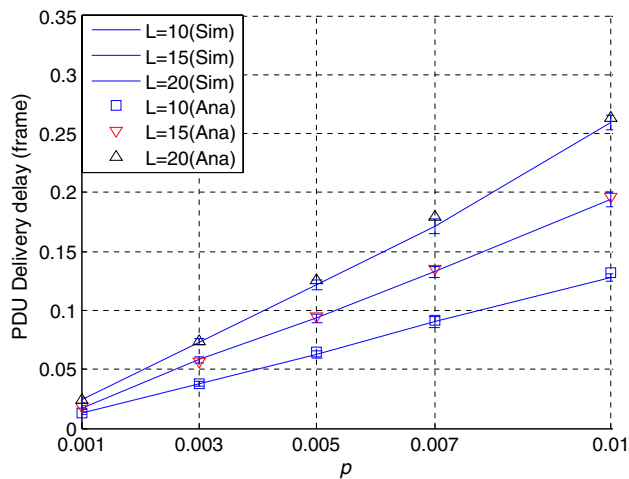


Fig. 13 PDU delivery delay versus p

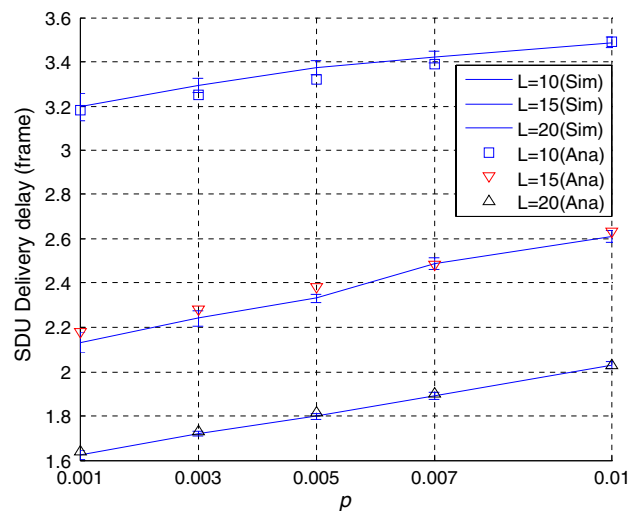


Fig. 14 SDU delivery delay versus p

that a larger p leads to a longer delivery delay. With the increase of p , the number of PDUs successfully transmitted at each time decreases. The delivery time required to successfully transmit a PDU and an SDU therefore increase. Meanwhile, the achieved goodput decreases with the increase of p , as shown in Fig. 15.

From Figs. 7–15, it can be seen that the simulation results match with the analytical ones very well. The gaps between the simulation result and the analytical results are less than 5%. The gaps can be further reduced if larger number of simulation runs is conducted.

6 Conclusions

We have studied the performance of the cumulative ARQ in IEEE 802.16 networks by jointly considering a scheduling scheme. An analytical model has been developed to

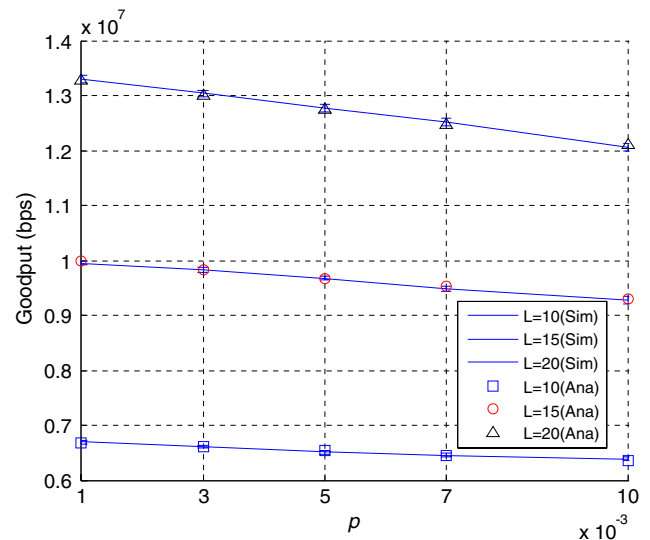


Fig. 15 Goodput versus p

investigate important performance metrics such as delivery delay and goodput, and the impact of different parameters on these performance metrics. Extensive simulation and numerical results have been given to validate the analytical model, which can provide meaningful guidelines for the selection of the operational parameters in the system design, such as the number of assigned bandwidth in each transmission chance (L) and the number of SSs selected for service in each MAC frame (h). Our further research on the selective ARQ in IEEE 802.16 networks is under way.

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Appendix: Analyses of service probability and inter-service time for SSs

In the Appendix, we discuss the service probability and inter-service time for SSs given that the general scheduling scheme proposed in [19] is adopted.

Analysis of service probability

The service probability is defined as the probability that an SS obtains the transmission opportunities at a DL sub-frame. Let the SS under discussion be referred to as the tagged SS. In order to analyze the service probability for the tagged SS, Firstly, we classify all SSs into three groups based on the channel state of the tagged SS at a specific MAC frame. Given the channel state of the tagged SS is at state ' n ', the three groups is composed of the SSs with channel conditions better than, same as, and worse than the

state ‘ n ’, respectively, which is denoted as the group G1, G2, and G3, respectively. Let k_1 , k_2 and k_3 denote the number of SSs in the group G1, G2, and G3, respectively. The tagged SS, which belongs to G2, obtains the chance of transmission only when the condition $k_1 < h$ holds. Otherwise all the selected SSs should come from the group G1. When the condition $k_1 < h$ is satisfied, the probability that the tagged SS obtains the chance of transmission is derived based on the value of k_1 and k_2 .

Since the total number of selected SSs is h , and k_1 SSs are at the channel state better than state ‘ n ’, $h - k_1$ is the quota left for the SSs at G2 and G3. When the left quota, $h - k_1$, is larger than k_2 , all the SSs at the group G2 are selected. That is, the tagged queue obtains the chance of transmission at this DL sub-frame with a probability 1. On the other hand, when $k_2 > h - k_1$, a tie occurs. In this case, the BS randomly selects $(h - k_1)$ out of k_2 SSs in G2. Therefore, the tagged queue obtains the chance of transmission with a probability $\left(\frac{h-k_1}{k_2}\right)$. To take these situations into account, we define the function $\xi(\cdot)$, as given in (25). It is concluded that k_1 is a value between the set $[0, h - 1]$ such that the tagged SS can obtain the chance of transmission. Under a specific value of k_1 , k_2 is in the set of $[1, M - k_1]$, where M is the total number of SSs in the system. The set begins with 1 since at least the tagged SS is in the group G2. When the values of k_1 and k_2 are given, k_3 is $M - k_1 - k_2$.

$$\xi\left(\frac{h-k_1}{k_2}\right) = \begin{cases} 1 & h - k_1 \geq k_2 \\ \frac{h-k_1}{k_2} & h - k_1 < k_2 \end{cases} \quad (25)$$

Let $\Omega(j_1)$, $\Omega(j_2)$, and $\Omega(j_3)$ denote the set of SSs in groups G1, G2 and G3, respectively. Thus, the probability that the tagged SS obtains the chance of service and stays at the state “ n ” along with the specific $\Omega(j_1)$, $\Omega(j_2)$ and $\Omega(j_3)$ can be expressed as

$$\xi\left(\frac{h-k_1}{k_2}\right) \left[\prod_{i_1 \in \Omega(j_1)} Pr(S_{i_1} > n) \prod_{i_2 \in \Omega(j_2)} Pr(S_{i_2} = n) \prod_{i_3 \in \Omega(j_3)} Pr(S_{i_3} < n) \right] \quad (26)$$

where the function $\xi(\cdot)$ is defined as (25), while

$$\prod_{i_1 \in \Omega(j_1)} Pr(S_{i_1} > n), \quad \prod_{i_2 \in \Omega(j_2)} Pr(S_{i_2} = n), \quad \text{and} \quad \prod_{i_3 \in \Omega(j_3)} Pr$$

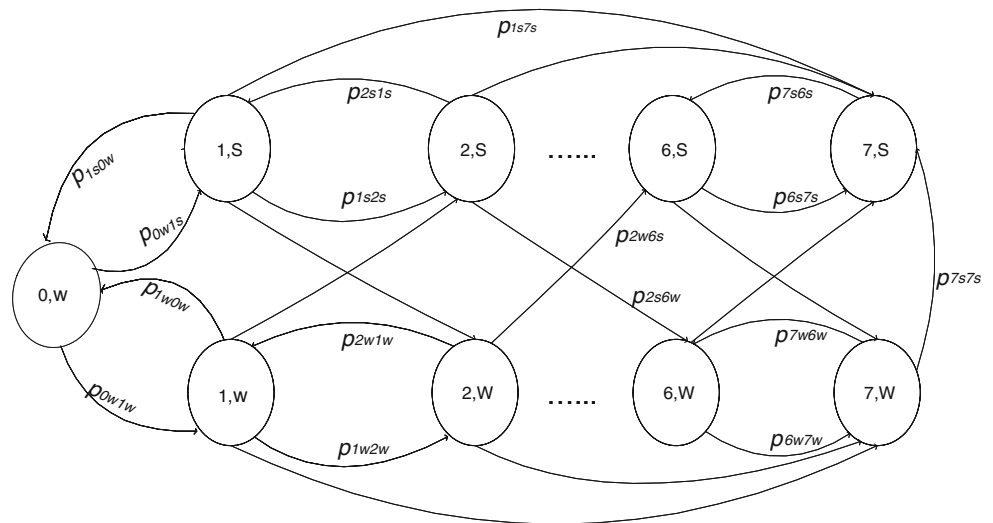
$(S_{i_3} < n)$ denote the probability that all SSs in groups G1, G2, and G3 are at the channel states better than, same as, and worse than the state n , respectively. For instance, the system is composed of SS1, SS2, SS3, SS4, SS5, and SS6. The tagged SS is SS1, which stays at channel state $n = 3$. Let $h = 3$, $k_1 = 2$, and $k_2 = 3$. The probability that SS1 obtains the chance of transmission with the channel state 3 while $\Omega(j_1)$, $\Omega(j_2)$, and $\Omega(j_3)$ are $\{SS2, SS3\}$, $\{SS1, SS4, SS5\}$, and $\{SS6\}$, respectively, is given by

$$\frac{1}{3} \left[\prod_{i_1 \in \{SS2, SS3\}} Pr(S_{i_1} > 3) \prod_{i_2 \in \{SS4, SS5\}} Pr(S_{i_2} = 3) \prod_{i_3 \in \{SS6\}} Pr(S_{i_3} < 3) \right] \quad (27)$$

Equation 27 is for the specific $\Omega(j_1)$, $\Omega(j_2)$, and $\Omega(j_3)$. In the following, the number of all possible $\Omega(j_1)$, $\Omega(j_2)$ and $\Omega(j_3)$ are taken into consideration. Let a_1 represents the number of possible combinations for selecting k_1 SSs out of $(M - 1)$ SSs to construct the group G1, where M is the total number of SSs in the system. After SSs in G1 are selected, there are $M - k_1$ SSs left. Let a_2 represent the number of possible combinations for selecting $(k_2 - 1)$ SSs out of the left $(M - k_1 - 1)$ SSs to construct the group G2. At last, the left $M - k_1 - k_2$ SSs consist of the group G3. We have $a_1 = \binom{M-1}{k_1}$ and $a_2 = \binom{M-k_2-1}{k_2-1}$. In other words, given a k_1 , the total number of possible $\Omega(j_1)$ is a_1 , and the set of all possible $\Omega(j_1)$ is represented by $\{\Omega(j_1), j_1 = 1, 2, \dots, a_1\}$. Given a $\Omega(j_1)$, the total number of possible $\Omega(j_2)$ is a_2 , and the set of all possible $\Omega(j_2)$ is represented by $\{\Omega(j_2), j_2 = 1, 2, \dots, a_2\}$. Note that given a $\Omega(j_1)$ and $\Omega(j_2)$, the number of possible $\Omega(j_3)$ is 1 since the group G3 is composed of all the left SSs that belong to neither G2 nor G3. In other words, j_3 is always 1. Let the vector $\Xi = \sigma_{S0} \sigma_{S1} \sigma_{S2} \sigma_{S3} \dots \sigma_{S7}$ be the probability for the tagged SS to obtain the transmission opportunity when the channel state of the tagged SS is ‘ n ’ ($n = 0, 1, \dots, 7$). The service probability of the tagged SS with the channel state ‘ n ’ is given as

$$\sigma_{s_n} = \begin{cases} \sum_{k_1=1}^{h-1} \xi\left(\frac{h-k_1}{k_2}\right) \sum_{j_1=1}^{a_1} \prod_{i_1 \in \Omega(j_1)} Pr(S_{i_1} > n) \sum_{k_2=1}^{M-k_1} \sum_{j_2=1}^{a_2} \prod_{i_2 \in \Omega(j_2)} Pr(S_{i_2} = n) \prod_{i_3 \in \Omega(j_3)} Pr(S_{i_3} < n) & n = 1, \dots, 7 \\ 0 & n = 0 \end{cases} \quad (28)$$

Fig. 16 The Markov model for the tagged SS



Note that $\sigma_{s_0} = 0$ since the tagged queue is not allowed to transmit when the channel state of the tagged SS is ‘0’, considering a higher error bit rate at such a poor channel condition.

Analysis of the inter-service time

By jointly considering ARQ and the general scheduling scheme, a Markov model can be constructed according to the channel states of the tagged SS. Each state in the Markov model represents the current channel state of the tagged SS and whether the tagged queue obtains the chance of transmission in the current DL sub-frame. Since the channel state includes $(N + 1)$ states ($N = 7$ in the study), and each state, excluding state ‘0’, may represent either one of the following two situations: the tagged SS either is selected or not, which equivalently means that the tagged queue either obtains or losses the chance of transmission at the current DL sub-frame; when the channel state of the tagged SS is at the state “0”, the tagged queue keeps silent, which equivalently means that the tagged queue always lose the chance of transmission. Thus, the combined Markov model consists of $2N + 1$ states as shown in Fig. 16, where (n, s) and (n, w) represent that the tagged queue obtains and loses the chance of transmission with the channel state of ‘ n ’, respectively.

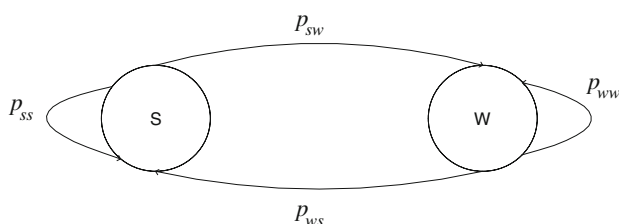


Fig. 17 The grouped Markov model for the tagged queue

The transmission probability matrix is given by

$$\underline{Q} = \begin{bmatrix} p_{00} & p_{01} & p_{01} & \dots & p_{0N} & p_{0N} \\ p_{10} & p_{11} & p_{11} & \dots & p_{1N} & p_{1N} \\ p_{10} & p_{11} & p_{11} & \dots & p_{1N} & p_{1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{N0} & p_{N1} & p_{N1} & \dots & p_{NN} & p_{NN} \\ p_{N0} & p_{N1} & p_{N1} & \dots & p_{NN} & p_{NN} \\ 1 - \sigma_{s_0} & 0 & 0 \dots & 0 & 0 & 0 \\ 0 & \sigma_{s_1} & 0 \dots & 0 & 0 & 0 \\ 0 & 0 & 1 - \sigma_{s_1} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \sigma_{s_N} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - \sigma_{s_N} & 0 \end{bmatrix} \quad (29)$$

In order to derive the inter-service time between two adjacent transmissions, states in Fig. 16 can be grouped into two states shown in Fig. 17, denoted as ‘S’ and ‘W’, respectively. The states ‘S’ and ‘W’ represent the states where the tagged queue obtains and losses the chance of transmission, respectively.

The transition probabilities of the grouped Markov model are given by

$$p_{sw} = \frac{\sum_{n=0}^7 \left[\theta(n, s) \sum_{j=0}^7 p_{ns, jw} \right]}{\sum_{n=0}^7 \theta(n, s)}; \quad p_{ss} = 1 - p_{sw} \quad (30)$$

$$p_{ws} = \frac{\sum_{n=0}^7 \left[\theta(n, w) \sum_{j=0}^7 p_{nw,js} \right]}{\sum_{n=0}^7 \theta(n, w)}; \quad p_{ww} = 1 - p_{ws} \quad (31)$$

where $\theta(n, s)$ is the steady state probability of the state (n, s) , and $p_{ns,jw}$ is the one-step transition probability from the state (n, s) to the state (j, w) ($n, j = 1, 2, \dots, 7$).

Let m denote the inter-service time, which is defined as the duration (in unit of frame) between two successive transmission chances of the tagged SS. The probability mass function of m , is given by

$$pr[m = i] = \begin{cases} p_{ss} & i = 0 \\ p_{sw}(p_{ww})^{i-1}p_{ws} & i > 0 \end{cases} \quad (32)$$

The mean of m is given by

$$\begin{aligned} E[m] &= \sum_{i=1}^{\infty} i p_{sw}(p_{ww})^{i-1}p_{ws} \\ &= p_{sw}p_{ws} \sum_{i=1}^{\infty} i (p_{ww})^{i-1} = \frac{p_{sw}}{p_{ws}} \end{aligned} \quad (33)$$

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Author Biographies

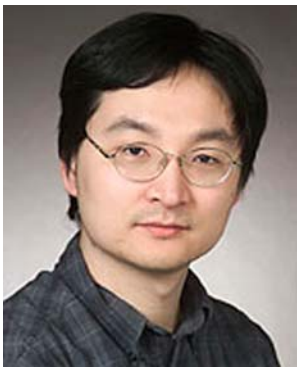


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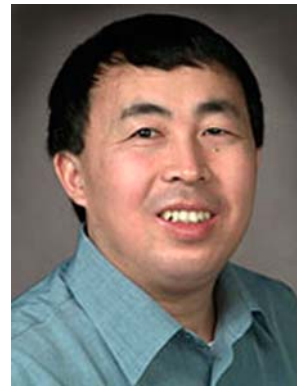
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