

# Lifetime and Energy Hole Evolution Analysis in Data-Gathering Wireless Sensor Networks

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**Abstract**—Network lifetime is a crucial performance metric to evaluate data-gathering wireless sensor networks (WSNs) where battery-powered sensor nodes periodically sense the environment and forward collected samples to a sink node. In this paper, we propose an analytic model to estimate the entire network lifetime from network initialization until it is completely disabled, and determine the boundary of energy hole in a data-gathering WSN. Specifically, we theoretically estimate the traffic load, energy consumption, and lifetime of sensor nodes during the entire network lifetime. Furthermore, we investigate the temporal and spatial evolution of energy hole, and apply our analytical results to WSN routing in order to balance the energy consumption and improve the network lifetime. Extensive simulation results are provided to demonstrate the validity of the proposed analytic model in estimating the network lifetime and energy hole evolution process.

**Index Terms**—wireless sensor network, network lifetime, energy hole, energy efficiency, routing.

## I. INTRODUCTION

WIRELESS sensor networks (WSNs), which are capable of sensing, computing, and wireless communication [1]–[3], are widely applied to many applications such as military surveillance, environmental monitoring, infrastructure and facility diagnosis, and other industry applications [4]. A data-gathering WSN consists of a large number of battery-powered sensor nodes that sense the monitored area and periodically send the sensing results to the sink. Since the battery-powered sensor nodes are constrained in energy resource and generally deployed in unattended hostile environment, it is crucial to prolong the network lifetime of WSN. Meanwhile, as energy consumption is exponentially increased with the communication distance according to the energy consumption model [5], multi-hop communication is beneficial to data gathering for energy conservation. However, since the nodes close to the sink should forward the data packets from other nodes, they exhaust their energy quickly, leading to an energy hole around the sink. As a result, the entire network is subject to premature death because it is separated by the energy hole.

There have been several existing works studying the energy consumption and network lifetime analysis for WSNs. Most of

them [6]–[8] focus on the duration from network initialization to the time when the first node dies (i.e., First Node Died Time, FNDDT), aiming to improve the network performances and optimize the FNDDT. Chen et al. [6] propose an analytic model for estimating the traffic load of sensor nodes and FNDDT in a multi-hop WSN. General network lifetime and cost models are also discussed in [9] to evaluate node deployment strategies. Since network lifetime is limited by unbalanced energy consumption, Ok et al. [10] propose a distributed energy balanced routing (DEBR) algorithm to balance the data traffic of sensor networks and consequently prolong the FNDDT. As hierarchical routing has been proved to be beneficial for network performance [11], especially for the scalability and energy consumption, research works also study the FNDDT of cluster-based WSNs. Lee et al. [11] derive the upper bound of FNDDT in cluster-based networks and investigate the effects of the number of clusters and spatial correlation on this bound. Liu et al. [12] also discuss the FNDDT of cluster-based networks, and propose a routing protocol to improve FNDDT based on unequal cluster radii.

Although most of existing works are effective to estimate FNDDT, the period from FNDDT to the time when all the sensor nodes are dead or the network is completely disabled (i.e., All Node Died Time, ANDDT) is relatively long [13]. For most applications, a small portion of dead nodes may not cause a network failure, although they can impact the network performances [12]–[14]. Thus, this period is non-negligible for the entire network lifetime. On the other hand, performance analysis on this period is difficult and intractable because the network is unstable after a few nodes die. Once the nodes with heavy load die, some other nodes should relay the data originally forwarded by these dead nodes. It leads to dynamical changes of the routing paths, as well as the traffic load of sensor nodes. Therefore, it is necessary to analyze the performance and network lifetime after FNDDT.

Recently, increasing attention has been paid to the period from FNDDT to ANDDT and analyzing the entire network lifetime [7], [13]–[15]. Ozgovde et al. [13] highlight that FNDDT is only an important stage of the entire network lifetime and ANDDT is an important factor to evaluate the network performance. To this end, they propose a utility based lifetime measurement framework called Weighted Cumulative Operational Time (WCOT), which calculates a network lifetime function based on the complete history of the network states. Lee et al. [15] analyze the entire aging process of the sensor network in a data-gathering WSN. However, they mainly focus on the network connection time rather than analyzing the

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energy consumption and lifetime of the sensor nodes. In [16], Li presents an annuli-based analytic model to analyze the network lifetime for data-gathering WSNs. The network is divided into a number of annulus whose widths are equal to the transmission range of the sensor nodes, and all the sensor nodes in the same annuli are assumed to die simultaneously. The network lifetime is defined as the time elapsing from network initialization until the sensor nodes in any one annuli are dead, and it is finally derived as a function of the number of annuli. Based on [16], Liu et al. [17] adopt an improved annuli-based analytic model to study FNDT and ANDT, and propose a non-uniform node distribution scheme to achieve optimal network lifetime. Meanwhile, some research works also investigate the entire network lifetime for event-driven WSNs [14], [18], [19]. Since data traffic in event-driven WSNs is bursty and follows a specific distribution, network lifetime is generally analyzed by probabilistic approaches. However, few of the existing works provide a comprehensive analysis for the energy consumption and lifetime of sensor nodes, and considers the negative impacts of energy hole on the network lifetime.

Energy hole is crucial and challenging for lifetime analysis in WSNs, because it can lead to a premature death of the network [20]. Olariu et al. [21] first prove that the energy hole problem is inevitable in the WSN under some specific conditions. Perillo et al. [22] analyze in what condition the energy holes could appear. Rahim et. al. [23] discuss the load balancing techniques to mitigate energy hole problem in large-scale WSNs, and propose a distributed heuristic solution to balance the energy consumption of sensor nodes by adjusting their transmission power. The energy hole problem has also been studied in cluster-based WSNs [24], [25]. Most of the existing works [25]–[27] suppose that energy hole locates around the sink, and design energy-efficient routing protocols to mitigate the unbalanced energy consumption and prolong the network lifetime. However, recent investigations [12], [17] point out that energy hole does not always emerge close to the sink and highly depends on some network parameters, such as the energy consumption model and transmission range of sensor nodes. However, theoretic analysis is not provided in existing works to estimate the emerging time and location of the energy hole, as well as its evolution process.

In this paper, we propose an analytic model to estimate the entire network lifetime from network initialization until it is completely disabled, and determine the boundary of energy hole in data-gathering WSNs. To accurately estimate the energy consumption of sensor nodes, we consider the energy consumption not only for data transmitting and receiving, but also for idle listening. Specifically, our contributions are three-fold.

(i) We propose an analytic model to estimate the traffic load, energy consumption and lifetime of sensor nodes during the entire network lifetime. Furthermore, we estimate the network lifetime under a given percentage of dead nodes, and the remaining energy of the network based on our analytical results. Extensive simulations demonstrate that the proposed analytic model can estimate the network lifetime within an error rate smaller than 5%.

(ii) Based on the lifetime analysis of sensor nodes, we investigate the temporal and spatial evolution of energy hole from emerging to partitioning the network, which provides a theoretical basis to mitigate or even avoid energy hole in WSNs.

(iii) To validate the effectiveness of our analytical results in guiding the WSN design, we apply them to WSN routing. The improved routing scheme based on our analytical results efficiently balances the energy consumption and significantly improves the network lifetime, including FNDT and ANDT.

The remainder of the paper is organized as follows. Section II introduces the system model and formulates our problem mathematically. In Section III, we theoretically analyze the traffic load, energy consumption and lifetime of sensor nodes. We determine the boundary of energy hole in Section IV as well as some observations on network characteristics. Section V validates the analytic model by comparing the analytical results with extensive simulation results. We apply our analytical results to WSN routing in Section VI. Finally, Section VII concludes the paper and outlines the future work.

## II. NETWORK MODEL AND PROBLEM STATEMENT

### A. Network Model

Consider a data-gathering WSN [6], [12], where  $n$  homogeneous sensors are randomly deployed in a circular region with the sink (base station) located at the centre [28], [29]. The network radius is  $R$  and the transmission range of each sensor is  $r$ . The sensor nodes are uniformly distributed in the network with a node density  $\rho$  [12], [14], [21]. Each sensor monitors a specific area and periodically sends the sensed data to the sink in a data period (or data round). Therefore, network lifetime can be measured by the number of data periods (or rounds). All the sensed data are delivered to the sink using greedy geographic routing [21], [24]. Sensor nodes forward packets to one of their neighboring nodes, which are geographically closest to the sink among all the neighbors. Geographic routing is scalable for large WSNs, since it only requires local information to make forwarding decisions. This routing scheme has been widely adopted in multi-hop wireless sensor and ad-hoc networks [11], [17]. In addition, our network is based on a collision-free MAC protocol without data loss just as the assumptions in [6], [12], [30], then we can focus on the impact on the network lifetime caused by the routing protocol, to provide a significant guidance for routing design on the network layer.

Sensors operate in active mode or sleep mode. The ratio of the time in active mode to a total data period is called duty cycle, denoted by  $\gamma$ . In general, sensors consume energy mainly in data receiving and transmitting, and idle listening when they are in active mode [31], [32]. We do not consider the energy consumption in sleep mode because it is small enough to be neglected [6], [32], [33]. According to the radio model [11], energy consumption for transmitting and receiving  $c$  bits of data are shown in Eq. (1) and Eq. (2), respectively.

$$E_t = \begin{cases} cE_{elec} + c\mathcal{E}_{fs}d^2, & d \leq d_0, \\ cE_{elec} + c\mathcal{E}_{amp}d^4, & d > d_0. \end{cases} \quad (1)$$

$$E_r = cE_{elec}. \quad (2)$$

where  $E_{elec}$  denotes transmitting circuit loss and  $d_0$  is the threshold distance. The free space channel model and the multi-path fading channel model are used in Eq. (1), according to the distance between the transmitter and receiver. If the transmission distance  $d$  is larger than the threshold distance  $d_0$ , multi-path fading channel model should be adopted; otherwise, free space channel model would be used.  $\mathcal{E}_{fs}$  and  $\mathcal{E}_{amp}$  denote the energy for power amplification in the two models, respectively.  $B$  is the data transmission rate of each sensor node. For idle listening, the energy consumption rate of the sensor nodes is denoted by  $E_{idle}$ .

### B. Problem Statement

We define the network lifetime as the duration from the network initialization to the time when the network is disabled. For a data-gathering WSN, the network is generally disabled under the following two situations. One is that all the sensor nodes exhaust their energy and die. The other is that the sink can not receive any data in a data period due to the energy hole partitioning the network, even if there are still a large number of alive nodes in the outer region of the energy hole. For simplification, we denote the network lifetime as ANDT and the time when the first node dies as FNDDT.

We describe the entire process of network lifetime in Fig. 1. Since the sensor nodes periodically send the sensed data to the sink in a data period, the network lifetime is slotted into a large number of data periods. We call the data periods in which at least a sensor node dies as death periods. Since sensor nodes die successively through the network lifetime, we can set there are  $k$  ( $k \leq n$ ) death periods  $[DP_0, DP_1, DP_2, \dots, DP_{k-1}]$  in the entire network lifetime. Therefore, the entire network lifetime is divided into  $k + 1$  stages  $[S_0, S_1, S_2, \dots, S_{k-1}, S_k]$  by the  $k$  death periods.  $S_i$  denotes the  $i$ -th network stage where the last data period is the  $i$ -th death period, e.g., the first sensor node dies at the end of the stage  $S_0$  and the network is totally disabled at the stage  $S_k$ . The number of alive nodes at each stage before the death period is denoted by  $[|S_0|, |S_1|, |S_2|, \dots, |S_{k-1}|, |S_k|]$  (e.g.,  $|S_0| = n$ ,  $|S_k| = 0$ ), and  $\{\{S_0\}, \{S_1\}, \{S_2\}, \dots, \{S_{k-1}\}, \{S_k\}\}$  denotes the sets of the alive nodes (e.g.,  $\{S_0\}$  is the set of all sensor nodes,  $\{S_k\} = \emptyset$  (i.e., empty set)). The duration at each stage, namely the number of data periods at each stage, is denoted by  $[l_0, l_1, l_2, \dots, l_{k-1}]$ . Thus,  $l^{(0)}$  is the network lifetime from network initialization until the first node dies (FNDDT). The average traffic load of node  $j$  in a data round of each stage is denoted by  $[p_j^{(0)}, p_j^{(1)}, \dots, p_j^{(i)}, \dots, p_j^{(k-1)}]$ . Obviously,  $p_j^{(k)} = 0$ . The transfer function from the traffic load to energy consumption is  $f$ , which can be determined according to the energy consumption model. Thus, the average energy consumption of node  $j$  in a data round of the  $i$ -th stage is  $p_j^{(i)} \xrightarrow{f} e_j^{(i)}$ . Similarly, we have  $e_j^{(k)} = 0$ .

The frequently used mathematical notations in this paper are summarized in Table I. The notations defined above can denote different network lifetimes. For example, FNDDT is  $l^{(0)}$

TABLE I  
FREQUENTLY USED NOTATIONS

Notation	Definition
$R$	Network radius (m)
$r$	Transmission range of sensor (m)
$\rho$	Density of sensor nodes
$E_0$	Initial energy of a sensor node
$B$	Data transmission rate of a sensor node
$d_0$	Threshold distance in radio model
$A_x$	A small region where the nodes' distances to the sink are equal or close to $x$
$DP_i$	The $i$ -th death period
$S_i$	The $i$ -th network stage
$\{S_i\}$	The set of alive nodes at the beginning of $S_i$
$ S_i $	The number of alive nodes at $S_i$
$p_j^{(i)}$	Per-round traffic load of node $j$ at $S_i$
$e_j^{(i)}$	Per-round energy consumption of node $j$ at $S_i$ (nJ)
$l^{(i)}$	The duration of the $i$ -th stage (rounds)

and ANDT is  $\sum_{i=0}^{k-1} l^{(i)}$ , and the network lifetime when half of the sensor nodes die is  $\sum_{j=0}^i l^{(i)}$  where  $|S_i| \cong n/2$ .

The objective of this paper is to estimate the nodal traffic load, energy consumption and network lifetime for a given network, so as to provide important guidelines for network optimization, such as routing design and node deployment. Specifically, we present our objectives as follows.

(1) For a given network, the average traffic load and energy consumption of the sensor nodes at each stage, that is, for each  $0 < i \leq n$  and  $0 \leq i \leq k - 1$ ,  $p_j^{(i)}$ ,  $e_j^{(i)}$ , as well as the energy transfer function  $f$  and the duration vector of the network stages  $[l_0, l_1, l_2, \dots, l_{k-1}]$  should be provided by our analysis. Then, we can describe the characteristics of the traffic load, energy consumption and lifetime over the entire network lifetime.

(2) The boundary and emerging time of the energy hole should be determined to provide a theoretical foundation for mitigating even avoiding the energy hole problem.

### III. ESTIMATION ON NODAL TRAFFIC LOAD, ENERGY CONSUMPTION, AND NETWORK LIFETIME CHARACTERISTICS

In this section, we theoretically estimate the traffic load and energy consumption of sensor nodes, as well as the duration of each network stage based on our system model.

The main idea of the analytic model can be described as follows. We first divide the network into a number of small regions where the nodes have similar distances to the sink. Since the energy consumption of the sensor nodes in the same region (i.e., with the similar distances to the sink) should be the same from a statistical point of view, we use the average energy consumption of this region as the nodal energy consumption of this region. Fig. 2 shows a sector zone of the network, where  $A_x$  is a region with the width of  $\varepsilon$

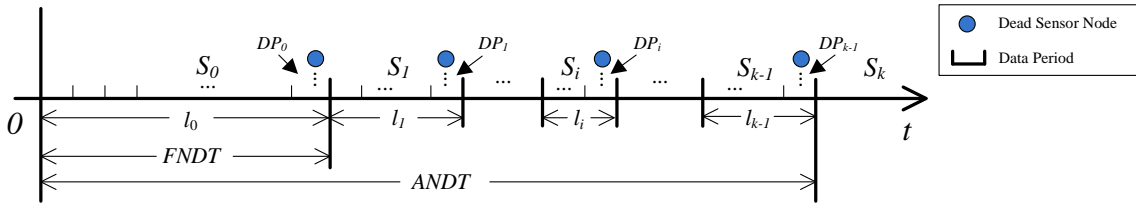


Fig. 1. Description of the entire process of network lifetime.

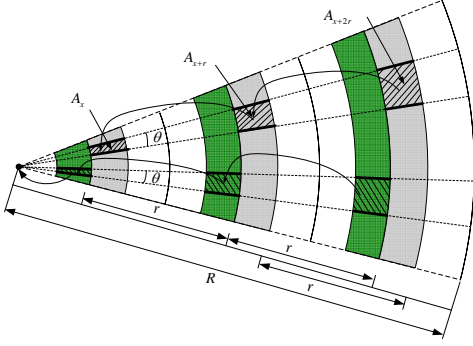


Fig. 2. Data forwarding model.

and  $\theta$  is the angle formed by  $A_x$  and the sink. The nodes' distances to the sink in  $A_x$  equal or are close to  $x$ . Since the nodal transmission radius is  $r$ ,  $A_x$  is supposed to forward the data from  $A_{x+r}$  whose distance to  $A_x$  is  $r$ . Likewise,  $A_{x+r}$  relays the data from  $A_{x+2r}$ . However, if the divided region is relatively large (i.e.,  $\varepsilon$  is relatively large), the energy consumption of the sensor nodes in it cannot be balanced. On the other hand, if the divided region is too small, there might be no node in this region. Therefore, we specify the constraint of  $\varepsilon$  and  $\theta$  as follows.

According to the network model, sensor nodes are uniformly distributed with a node density  $\rho$ . Since  $A_x$  is a relatively small region, if  $x \geq \varepsilon$ , the area of  $A_x$  can be approximated as  $W_{A_x} = x\varepsilon\theta$ ; otherwise,  $W_{A_x} = \theta\varepsilon^2$ . Then, the number of nodes in  $A_x$  is

$$N_{A_x} = W_{A_x} \cdot \rho = \begin{cases} x\varepsilon\theta\rho, & \text{if } x \geq \varepsilon; \\ \theta\varepsilon^2\rho, & \text{otherwise.} \end{cases} \quad (3)$$

As we discussed above, the number of nodes in  $A_x$  should be larger than 1, which means  $N_{A_x} \geq 1$ . That is,

$$\varepsilon \geq \frac{1}{\sqrt{\theta\rho}}. \quad (4)$$

Therefore, we can choose the value of  $\varepsilon$  and  $\theta$  under the constraint of Eq. (4) to keep the region of  $A_x$  small enough, which can ensure the balanced energy consumption of the sensor nodes in  $A_x$ .

#### A. Traffic Load Analysis at $S_0$

$S_0$  indicates a stage when no node dies, and hence is the most important stage with the best performance. We first analyze the traffic load of sensor nodes based on our analytic model described above, by the following theorem.

*Theorem 1:* Assume node  $j$  is in the small region of  $A_x$  with the width of  $\varepsilon$ . Denote  $x$  as the distance between  $A_x$  and the sink, and  $\theta$  as the angle formed by  $A_x$  and the sink. If each node generates one data packet per round, the average data amount sent by  $j$  in a round at  $S_0$  is:

$$p_j^{(0)} = \begin{cases} (z_1 + 1) + \frac{z_1(1 + z_1)r}{2x_1}, & \text{if } x \geq \varepsilon; \\ \frac{1}{2}(z_2 + 2)\varepsilon^2\theta\rho + \frac{1}{2}z_2(z_2 + 1)r\varepsilon\theta\rho, & \text{otherwise.} \end{cases}, \quad (5)$$

where  $z_1 = \lfloor (R - x)/r \rfloor$  and  $z_2 = \lfloor (R - \varepsilon)/r \rfloor$ .

*Proof:* Since node  $j$  is in the small region of  $A_x$ , its traffic load can be calculated as the average traffic load in  $A_x$  according to our analytic model. Therefore, we first calculate the average traffic load in  $A_x$ . As shown in Fig. 2,  $\varepsilon$  is the width of  $A_x$  and  $\theta$  denotes the angle formed by  $A_x$  and the sink, thus, we can obtain the number of nodes in  $A_x$  according to Eq. (3). As these nodes receive and forward the data from the upstream regions, the number of sensor nodes in the upstream regions  $A_{x+ir}$ , according to Eq. (3), is

$$N_{A_{x+ir}} = \begin{cases} (x + ir)\varepsilon\theta\rho & | 0 < i \leq z_1, & \text{if } x \geq \varepsilon; \\ (\varepsilon/2 + ir)\varepsilon\theta\rho & | 0 < i \leq z_2, & \text{otherwise.} \end{cases}, \quad (6)$$

where  $z_1 = \lfloor (R - x)/r \rfloor$  and  $z_2 = \lfloor (R - \varepsilon)/r \rfloor$ .

Since each node only generates a data packet per round, the number of data packets equals to the number of the involved nodes. Thus, the number of data packets sent by  $A_x$  is:

$$D_{A_x} = N_{A_x} + N_{A_{x+r}} + \dots + N_{A_{x+zr}}. \quad (7)$$

Based on Eq. (7), we have the average traffic load of  $A_x$  as  $\frac{D_{A_x}}{N_{A_x}}$ . Since the traffic load of the node  $j$  approximately equals the average traffic load of the sensor nodes in  $A_x$ , the traffic load of the node  $j$  at  $S_0$  should be  $p_j^{(0)} = \frac{D_{A_x}}{N_{A_x}}$ . With some simple calculation, we have  $p_j^{(0)}$  as Eq. (5). ■

#### B. Energy Consumption Analysis at $S_0$ and Estimation of $l^{(0)}$

The traffic load of sensor nodes at  $S_0$  can be determined by Thm. 1. If each data packet contains  $\tau$  bits, the total amount of transmitted data is  $p_x^{(0)}\tau$ . In this paper, energy consumption for network control is not considered since it is almost the same for each node and relatively small in greedy geographic routing [14], [17]. Therefore, we determine the energy consumption of sensor nodes at  $S_0$  in the following theorem.

*Theorem 2:* Denote  $T_r$  as the time period for one data gathering round. Node  $j$  is in the region  $A_x$ , where  $x$  is the distance between  $A_x$  and the sink. If the data transmission rate of the sensor node is  $B$  bits/second, in a data round, the average energy consumption  $e_j^{(0)}$  of  $j$  is  $e_j^{(0)} = e_{j,r}^{(0)} + e_{j,t}^{(0)} + e_{j,i}^{(0)}$ , where

$$\begin{cases} e_{j,r}^{(0)} = (p_x^{(0)} - 1)\tau E_{elec} \\ e_{j,t}^{(0)} = p_x^{(0)}\tau(E_{elec} + \varepsilon_\kappa d^\alpha) \\ e_{j,i}^{(0)} = E_{idle}t_{j,i}^{(0)} = E_{idle}(t_a - 2p_x^{(0)}\tau/B + \tau/B) \end{cases} \quad (8)$$

and if  $x > r$ ,  $d = r$ ; otherwise,  $d = x$ , and if  $d \leq d_0$ ,  $\varepsilon_\kappa = \varepsilon_{fs}$  and  $\alpha = 2$ ; otherwise,  $\varepsilon_\kappa = \varepsilon_{amp}$  and  $\alpha = 4$ .

*Proof:* In a data round, the energy consumption of node  $j$  consists of the following three parts.

(a) Energy consumption for data receiving.

Since node  $j$  is in the region  $A_x$ , the received data amount in a round is  $(p_j^{(0)} - 1)$ , according to Thm. 1. Thus, the energy consumption for receiving is  $e_{j,r}^{(0)} = (p_j^{(0)} - 1)\tau E_{elec}$ .

(b) Energy consumption for data transmitting.

Since the data amount sent by  $j$  in a round is  $p_j^{(0)}$ , the energy consumption for data transmitting is

$$\begin{cases} e_{j,t}^{(0)} = p_j^{(0)}\tau(E_{elec} + \varepsilon_{fs}d^2), & \text{if } d \leq d_0 \\ e_{j,t}^{(0)} = p_j^{(0)}\tau(E_{elec} + \varepsilon_{amp}d^4), & \text{otherwise} \end{cases} \quad (9)$$

If  $x > r$ ,  $d = r$ ; otherwise  $d = x$ . That is because the transmission distance is  $r$  if  $x \geq r$ ; otherwise, it is  $x$ .

(c) Energy consumption for idle listening.

According to the network model, the duty cycle is  $\gamma$  [31], [32]. Thus, the active time per round is  $t_a = T_r\gamma$ . The energy consumption for idle listening is the multiplication of  $E_{idle}$  and the duration in idle listening. Since the duration in idle listening, denoted by  $t_{j,i}^{(0)}$ , is the active time excluding the time for data transmitting and receiving, we have  $t_{j,i}^{(0)} = t_a - (p_j^{(0)} - 1)\tau/B - p_j^{(0)}\tau/B$ . Therefore, we derive the energy consumption for idle listening as  $e_{j,i}^{(0)} = E_{idle}t_{j,i}^{(0)} = E_{idle}(t_a - 2p_x^{(0)}\tau/B + \tau/B)$ .

To summarize, in a round, the energy consumption  $e_j^{(0)}$  of node  $j$ , is  $e_j^{(0)} = e_{j,r}^{(0)} + e_{j,t}^{(0)} + e_{j,i}^{(0)}$ . ■

According to Thm. 1 and 2, several phenomena can be concluded as follows. (a) Nodal traffic load and energy consumption have a direct relationship with the transmission radius  $r$ , which might cause the location of the hotspot deviating from the adjacent area of the sink. (b) When  $r$  is fixed, the total energy consumption is impacted by the energy consumption for idle listening. (c) For the nodes near the sink, since the traffic load is relatively large, the time period for data transmitting and receiving is long, while the time period in idle listening is relatively short. Thus, the proportion of energy consumption for idle listening for these nodes is lower than the nodes far from the sink.

Since the first batch of dead nodes must be the ones with the maximum energy consumption in the network, the FNDDT  $l^{(0)}$  is

$$l^{(0)} = \left\lfloor \frac{E_0}{\max(e_x^{(0)})} \right\rfloor, \quad (10)$$

where  $E_0$  is the initial energy of the sensor nodes.

Therefore, we summarize the analytical results at  $S_0$  as follows.

(1) The per-round traffic load of sensor nodes at  $S_0$ , i.e.,  $[p_1^{(0)}, p_2^{(0)}, \dots, p_j^{(0)}, \dots, p_n^{(0)}]$ , can be obtained by Thm. 1.

(2) The per-round energy consumption of each node at  $S_0$ , i.e.,  $[e_1^{(0)}, e_2^{(0)}, \dots, e_j^{(0)}, \dots, e_n^{(0)}]$ , can be obtained by Thm. 2.

(3) The FNDDT  $l^{(0)}$  can be determined by the Eq. (10) and the energy transfer function  $f$  is the energy consumption formula as Eq. (8).

### C. Estimation on Traffic Load, Energy Consumption and Network Lifetime from $S_1$ to $S_{k-1}$

In the previous subsections, we have determined the traffic load and energy consumption of sensor nodes at  $S_0$ . In this subsection, we analyze the traffic load and energy consumption of the sensor nodes after  $S_0$ , which is complicated because network routing paths change dynamically after  $S_0$ .

At first, we should find out that which part of sensor nodes die first. According to Eq. (10), the sensor nodes with the maximum energy consumption will die first. And we can determine the location of the first batch of dead nodes by combining Eq. (5), (8) and (10). According to our analytic model, the energy consumption of the sensor nodes in a region with width of  $\varepsilon$  are the same and the energy consumption of the regions with the same distance to the sink should be the same too. Therefore, the network can be divided into a number of ring regions with the same energy consumption and the width of  $\varepsilon$ . Without loss of generality, we set the first batch of dead nodes in the ring region of  $[u, u + \varepsilon]$ , the number of dead nodes is  $(\pi(u + \varepsilon)^2 - \pi u^2) \cdot \rho$ .

Then, we should determine that where are the  $i$ -th ( $i \geq 2$ ) batch of dead nodes. The  $i$ -th ( $2 \leq i \leq k$ ) batch of dead nodes should die in  $DP_{i-1}$  (i.e., the last data period of  $S_{i-1}$ ). Based on the Thm. 1 and 2, we have Corollary 1.

*Corollary 1:* For  $2 \leq i \leq k - 1$  and  $1 \leq j \leq n$ , given the energy consumption of node  $j$  from  $S_0$  to  $S_{i-1}$  as  $[e_j^{(0)}, e_j^{(1)}, \dots, e_j^{(i-1)}]$ , and the duration of the network stages before  $S_{i-1}$  as  $[l^{(0)}, l^{(1)}, \dots, l^{(i-2)}]$ , we have

$$l^{(i-1)} = \min_{j \in \{S_{i-1}\}} \left( \left( E_0 - \sum_{w=0}^{i-2} e_j^{(w)} l^{(w)} \right) / e_j^{(i-1)} \right), \quad (11)$$

where  $\{S_{i-1}\}$  is the set of alive nodes at the beginning of  $S_{i-1}$ . And the set of the  $i$ -th batch of dead nodes are

$$n_{died} = \arg \min_{j \in \{S_{i-1}\}} \left( \left( E_0 - \sum_{w=0}^{i-2} e_j^{(w)} l^{(w)} \right) / e_j^{(i-1)} \right). \quad (12)$$

*Proof:* For each alive node  $j$  at  $S_{i-1}$ , the energy consumption of node  $j$  from  $S_0$  to  $S_{i-1}$  is  $[e_j^{(0)}, e_j^{(1)}, \dots, e_j^{(i-1)}]$ , and the duration of the network stages before  $S_{i-1}$  is  $[l^{(0)}, l^{(1)}, \dots, l^{(i-2)}]$ . Therefore, the remaining energy of node  $j$  at the beginning of  $S_{i-1}$ , denoted by  $E_{j,remain}^{(i-1)}$ , is

$$E_{j,remain}^{(i-1)} = E_0 - \sum_{w=0}^{i-2} e_j^{(w)} l^{(w)}. \quad (13)$$

Since the energy consumption of node  $j$  at  $S_{i-1}$  is  $e_j^{(i-1)}$ ,  $j$ 's lifetime at  $S_{i-1}$  can be determined as

$$l_j^{(i-1)} = E_{j,remain} / e_j^{(i-1)}. \quad (14)$$

Therefore, for each alive node  $j$  at  $S_{i-1}$ , the duration of  $S_{i-1}$  should be the minimum  $l_j^{(i-1)}$  and the  $i$ -th ( $2 \leq i \leq k$ ) batch of dead nodes  $n_{died}$  should be the nodes that has the minimum  $l_j^{(i-1)}$ . We have,  $l^{(i-1)} = \min_{j \in \{S_{i-1}\}} (l_j^{(i-1)})$  and  $n_{died} = \arg \min_{j \in \{S_{i-1}\}} (l_j^{(i-1)})$ , where  $\{S_{i-1}\}$  denotes the set of alive nodes at the beginning of stage  $S_{i-1}$ . ■

Corollary 1 determines the duration of  $S_i$  based on the energy consumption of the sensor nodes at  $S_i$ . Next, we should analyze and determine the traffic load and energy consumption of the network after the  $i$ -th ( $1 \leq i \leq k-2$ ) batch of nodes die. According to our analytic model, the energy consumption of the sensor nodes in the regions with the same distances to the sink should be equal. Therefore, as we discussed above, we can set the  $i$ -th batch of dead nodes in the region  $[u_i, u_i + \varepsilon]$ . Then, three cases should be considered. (a) All the nodes around the region are alive. (b) The nodes close to the region, particularly on the side near the sink, are dead. (c) The nodes close to the region, particularly on the side far from the sink, are dead. We denote the continuous dead region including  $[u_i, u_i + \varepsilon]$  and the original dead region as  $[u_s, u_e]$ , i.e., all nodes in  $[u_s, u_e]$  are dead. In the first case,  $[u_s, u_e]$  is equal to  $[u_i, u_i + \varepsilon]$ . Thus, for the dead region  $[u_s, u_e]$ , the original data forwarding should be changed. Specifically, the traffic load of the following four regions is different from that at  $S_{i-1}$ .

(1) The traffic load of the sensor nodes in the dead region  $[u_s, u_e]$  is 0.

(2) The traffic load increases in the region  $[u_e, u_e + \varepsilon]$ . The data originally transmitted by the region  $[u_s, u_e]$  is forwarded by region  $[u_e, u_e + \varepsilon]$  now. In other words, the data of  $[u_s + r, u_e + r]$  that is supposed to be forwarded by the region  $[u_e, u_e + \varepsilon]$  leads to the increment of traffic load in  $[u_e, u_e + \varepsilon]$ .

(3) The traffic loads increase in the regions  $[u_e - ir, u_e - ir + \varepsilon] | 0 \leq i \leq z_3$ , where  $z_3 = \lfloor \frac{u_e}{r} \rfloor$ . Due to the increased traffic load in the region  $[u_e, u_e + \varepsilon]$ , the traffic loads in the corresponding downstream regions  $[u_e - ir, u_e - ir + \varepsilon] | 0 < i \leq z_3$  increase.

(4) If  $u_s > r$ , the traffic loads decrease in the regions  $[u_s - ir, u_e - ir] | 0 < i \leq z_4$ , where  $z_4 = \lfloor \frac{u_s}{r} \rfloor$ . Because the data of region  $[u_s, u_e]$  is originally forwarded by the regions  $[u_s - ir, u_e - ir] | 0 < i \leq z_4$ . Since the nodes in the region  $[u_s, u_e]$  are dead, the traffic load in these downstream regions should be decreased.

Except the regions discussed above, the traffic load in other regions stays the same. We summarize the traffic load changes in different regions of the network in Thm. 3.

**Theorem 3:** Let the  $i$ -th ( $1 \leq i \leq k$ ) batch of dead nodes be in the region  $[u_i, u_i + \varepsilon]$ , and  $[u_s, u_e]$  denote the continuous dead region including  $[u_i, u_i + \varepsilon]$  and the original dead region. After the  $i$ -th ( $1 \leq i \leq k$ ) batch of nodes die at  $S_{i-1}$ , for each alive node  $j$  in the region  $A_x$  where  $x$  is distance between  $A_x$

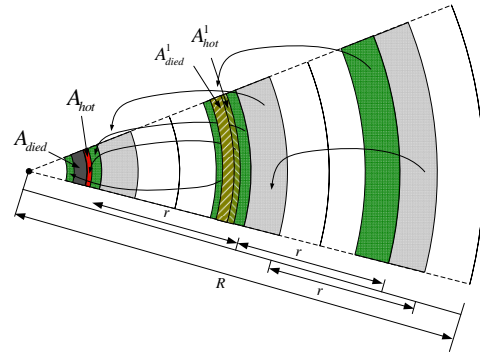


Fig. 3. The change of data forwarding after  $S_0$ .

and the sink, the traffic load at  $S_i$  changes to

$$p_j^{(i)} = \begin{cases} 0, & \text{if } x \in [u_s, u_e] \\ p_j^{(i-1)} + D_{died} / ((u_e - jr + \varepsilon)^2 - (u_e - jr)^2) \pi \rho & \text{if } x \in [u_e - jr, u_e - jr + \varepsilon] | 0 \leq j \leq z_3 \\ p_j^{(i-1)} - D_{died} / ((u_s - jr)^2 - (u_e - jr)^2) \pi \rho & \text{if } x \in [u_s - jr, u_e - jr] | 0 < j \leq z_4 \text{ and } u_s > r \\ p_j^{(i-1)}, & \text{otherwise} \end{cases}, \quad (15)$$

where  $D_{died} = f(\rho) \cdot \sum_{k=1}^z (\pi(u_e + kr)^2 - \pi(u_s + kr)^2)$ ,  $z = \lfloor \frac{R - u_s}{r} \rfloor$ ,  $z_3 = \lfloor \frac{u_e}{r} \rfloor$  and  $z_4 = \lfloor \frac{u_s}{r} \rfloor$ .

*Proof:* As shown in Fig. 3, denote the region  $[u_s, u_e]$  by  $A_{died}$ . It is obvious that the traffic load of the dead region  $A_{died}$  is 0. After the sensor nodes of  $A_{died}$  die, its corresponding upstream region  $A_{died}^1$  is supposed to forward data to the sink through the region  $A_{hot}$  which is close to  $A_{died}$  ( $A_{hot}$  is the region  $[u_e, u_e + \varepsilon]$ ). Thus,  $A_{hot}$  bears not only the data traffic of  $A_{hot}^1$ , but also the data traffic of  $A_{died}^1$  which is originally undertaken by  $A_{died}$ .

We first determine the traffic load of  $A_{died}^1$ . The traffic load of  $A_{died}^1$  consists of the data traffic in its region and from its upstream regions. The area of  $A_{died}^1$  is  $\pi(u_e + r)^2 - \pi(u_s + r)^2$ . And the area of the upstream regions of  $A_{died}^1$  are  $\pi(u_e + ir)^2 - \pi(u_s + ir)^2 | 1 \leq i \leq z$ , where  $z = \lfloor \frac{R - u_s}{r} \rfloor$ . Therefore, the traffic load of  $A_{died}^1$  is

$$D_{died} = \sum_{k=1}^z (\pi(u_e + kr)^2 - \pi(u_s + kr)^2) \cdot \rho. \quad (16)$$

After the nodes of  $A_{died}$  die, the data originally sent to this region is now transmitted to the nodes around  $A_{died}$ , leading to the data originally forwarded by  $A_{died}$  is now sent to  $A_{hot}$ . Therefore,  $A_{hot}$  and its corresponding downstream regions  $[u_e - jr, u_e - jr + \varepsilon] | 0 < j \leq z_3$ , where  $z_3 = \lfloor \frac{u_e}{r} \rfloor$ , should forward extra  $D_{died}$  data besides its original data.

Since the number of nodes in these regions is  $(\pi(u_e - jr + \varepsilon)^2 - \pi(u_e - jr)^2) \cdot \rho | 0 \leq j \leq z_3$ , and the increased traffic load of each node in these regions is  $D_{died} / ((\pi(u_e - jr + \varepsilon)^2 - \pi(u_e - jr)^2) \cdot \rho | 0 \leq j \leq z_3$ .

Similarly, if  $u_e > r$ , the data of this region has to be sent to

**Algorithm 1** Determining the traffic load, energy consumption and lifetime of sensor nodes at each network stage.

**Input:** Network radius  $R$ , transmission radius  $r$ , node density of the network  $\rho$ , and other parameters.

**Output:** For each stage  $i$  and each node  $j$ , return the nodal traffic load  $p_j^{(i)}$ , energy consumption  $e_j^{(i)}$ , as well as the energy transfer function  $f$  and lifetime vector  $l$ .

- 1: Determine the traffic load and energy consumption of each node at stage  $S_0$ , i.e.,  $[p_1^{(0)}, p_2^{(0)}, \dots, p_j^{(0)}, \dots, p_n^{(0)}]$  and  $[e_1^{(0)}, e_2^{(0)}, \dots, e_j^{(0)}, \dots, e_n^{(0)}]$ , according to Thm. 1 and 2;
- 2:  $i = 1$ ;
- 3: **while** the sink can receive data in a data period **do**
- 4: According to Corollary 1, calculate the lifetime  $l^{(i-1)}$  at stage  $S_{i-1}$ , and the  $i$ -th batch of dead nodes region  $[u_i, u_i + \varepsilon]$ ;
- 5: Determine the traffic load and energy consumption of the sensor nodes at stage  $S_i$ , i.e.,  $[p_1^{(i)}, p_2^{(i)}, \dots, p_j^{(i)}, \dots, p_n^{(i)}]$  and  $[e_1^{(i)}, e_2^{(i)}, \dots, e_j^{(i)}, \dots, e_n^{(i)}]$ , according to Thm. 3 and 4,;
- 6:  $i = i + 1$ ;
- 7: **end while**
- 8: **return** The traffic load and energy consumption  $p_j^{(i)}$  and  $e_j^{(i)}$  (for each  $i$  and  $j$ ), and the network stage duration vector  $l^{(i)}$  (for each  $i$ ).

the sink via more than one hop. Then, the downstream regions  $[u_s - jr, u_e - jr] | 0 < j \leq z_4$ , where  $z_4 = \lfloor \frac{u_s}{r} \rfloor$ , do not forward the data originally transmitted by  $A_{died}$ . Therefore, the decreased traffic load of each node in these regions is  $D_{died} / ((\pi(u_s - jr)^2 - \pi(u_e - jr)^2) \cdot \rho) | 0 < j \leq z_4$ .

Meanwhile, the traffic load of other regions should stay the same as that at  $S_{i-1}$ . Based on the analysis above, the theorem can be proved. ■

According to Thm. 3, the energy consumption of sensor nodes at  $S_i$  changes as the following theorem.

*Theorem 4:* Assume node  $j$  is in a region  $A_x$ , where  $x$  is the distance between  $A_x$  and the sink. After sensor nodes die at  $S_{i-1}$ , the per-round energy consumption of  $j$  at  $S_i$  changes to  $e_j^{(i)} = e_{x,r}^{(i)} + e_{x,t}^{(i)} + e_{x,i}^{(i)}$ , where

$$\begin{cases} e_{j,r}^{(i)} = (p_x^{(i)} - 1)\tau E_{elec} \\ e_{j,t}^{(i)} = p_x^{(i)}\tau(E_{elec} + \varepsilon_\kappa d^\alpha) \\ e_{j,i}^{(i)} = E_{idle}t_{j,i}^{(i)} = E_{idle}(t_a - 2p_x^{(i)}\tau/B + \tau/B) \end{cases} \quad (17)$$

and if  $x > r$ ,  $d = r$ ; otherwise,  $d = x$ , and if  $d \leq d_0$ ,  $\varepsilon_\kappa = \varepsilon_{fs}$  and  $\alpha = 2$ ; otherwise,  $\varepsilon_\kappa = \varepsilon_{amp}$  and  $\alpha = 4$ .

*Proof:* Similar with the proof of Thm. 2. ■

Based on the previous theorems, Alg. 1 shows how to determine the traffic load, energy consumption and lifetime of the sensor nodes at each network stage.

#### D. Analysis on Network Lifetime and Remaining Energy

Alg. 1 can determine the entire network lifetime and the duration of each network stage. However, different WSN

applications have different lifetime requirements. Intuitively, lifetime requirement can be described by the percentage of dead nodes in the network, which is also called death ratio. If we use  $l_{\%}^i$  to denote the required lifetime when the death ratio reaches  $w\%$ ,  $l_{\%}^0$  means FNDT and  $l_{\%}^{100}$  means ANDT. Based on our analytical results, we can have Corollary 2.

*Corollary 2:* Given a required death ratio  $w\%$ , the network lifetime  $l_{\%}^w$  is  $l_{\%}^w = \sum_{i=0}^{\sigma-1} l^{(i)}$ , where  $\frac{n - |S_\sigma|}{n} \geq w\%$  and  $\sigma = \arg \min_{1 \leq i \leq k} \left\{ \frac{n - |S_i|}{n} - w\% \right\}$ .

*Proof:* Since the number of the sensor nodes in the network is  $n$  and the number of alive nodes at the  $i$ -th network stage is  $|S_i|$ , the percentage of dead nodes at the  $i$ -th network stage is  $\frac{n - |S_i|}{n}$ .

If the required death ratio is  $w\%$ , it means the network lifetime consists of all the network stages where the percentage of dead nodes is below  $w\%$ . Therefore, we can determine the network stage  $\sigma$  where the percentage of dead nodes firstly exceeds  $w\%$ . According to our analysis above, we have  $\sigma = \arg \min_{1 \leq i \leq k} \left\{ \frac{n - |S_i|}{n} - w\% \right\}$ , where  $\frac{n - |S_\sigma|}{n} \geq w\%$ . And the network lifetime  $l_{\%}^w$  is the duration from  $S_0$  to  $S_{\sigma-1}$ , i.e.,  $l_{\%}^w = \sum_{i=0}^{\sigma-1} l^{(i)}$ . ■

Network lifetime can be determined by Corollary 2 under a given death ratio. It indicates the proposed analytic model can estimate the network lifetime for WSN applications with different lifetime requirements. When the percentage of dead nodes in the network exceeds the required death ratio, the network is considered as disabled and the remaining energy of the network becomes useless. Therefore, remaining energy can be a performance metric to evaluate the energy efficiency of WSNs, and the remaining energy of the sensor nodes can guide the WSN redeployment. We determine the remaining energy of sensor nodes in the following corollary.

*Corollary 3:* For each network stage  $S_i$ , the remaining energy of node  $j$  after  $S_i$  is  $\varphi_j^{(i)} = E_0 - \sum_{c=0}^i (e_j^{(c)} \cdot l^{(c)})$ , and the remaining energy of the network after  $S_i$  is  $\varphi^{(i)} = nE_0 - \sum_{c=0}^i \sum_{j=0}^n (e_j^{(c)} \cdot l^{(c)})$ .

*Proof:* Since  $e_j^{(i)}$  is the average energy consumption of node  $j$  in a data round at  $S_i$  and  $l^{(i)}$  is the duration of  $S_i$ , we can determine the energy consumption of node  $j$  during  $S_i$  as  $E_{j,i} = e_j^{(i)} \cdot l^{(i)}$ . From  $S_0$  to  $S_i$ , the total energy consumption of node  $j$  is  $E_{use}^{i,j} = \sum_{c=0}^i E_{i,j}$  and the total energy consumption of the network is  $E_{use}^i = \sum_{c=0}^i \sum_{j=0}^n E_{i,j}$ . As the initial energy of the network is  $nE_0$ , after  $S_i$ , the remaining energy of node  $j$  is  $\varphi_j^{(i)} = E_0 - E_{use}^{i,j} = E_0 - \sum_{c=0}^i (e_j^{(c)} \cdot l^{(c)})$  and the remaining energy of the network is  $\varphi^{(i)} = nE_0 - E_{use}^i = nE_0 - \sum_{c=0}^i \sum_{j=0}^n (e_j^{(c)} \cdot l^{(c)})$ . ■

**Algorithm 2** Determining the emerging time and boundary of the energy hole.

**Input:** Network radius  $R$ , transmission radius  $r$ , node density of the network  $\rho$ , and other parameters.

**Output:** The energy hole boundary  $[d_{shole}, d_{ehole}]$  and emerging time  $t_h$ .

- 1: Run Alg. 1 until there is a continuous dead ring whose width  $d$  satisfies  $d \geq r$ ;
- 2: The boundary of this dead region is the request  $[d_{shole}, d_{ehole}]$ ;
- 3: The lifetime at this network stage is the emerging time  $t_h$ ;
- 4: **return**  $[d_{shole}, d_{ehole}]$  and  $t_h$ .

#### IV. ENERGY HOLE AND NETWORK CHARACTERISTICS

##### A. Analysis on The Energy Hole Evolution

In this section, we investigate the temporal and spatial evolution of energy hole based on our analytical result. The traffic load and energy consumption of the sensor nodes and the network lifetime can be determined by Alg. 1, where the termination condition is that the sink cannot receive any data in a data period, which consists of two cases. One is all nodes die due to energy exhaustion. The other is some nodes still have remaining energy, but the sink is separated from the outer nodes after the formation of the energy hole. Thus, even if the network still has remaining energy, the network becomes useless and is also considered as disabled.

We can easily judge the algorithm is terminated in which case by checking if there are sensor nodes with remaining energy in the network. If it is the second case, the formation of energy hole should be analyzed temporally and spatially. According to our analytic model, at least one sensor node will die after each network stage. Since the location of the dead nodes can be determined by Alg. 1, we can check if the dead sensor nodes form a continuous dead ring with the width  $d$  and  $d \geq r$  after each network stage. The network may be partitioned by the continuous dead ring, which is exactly the energy hole of the network. Alg. 2 shows how to determine the emerging time and boundary of the energy hole.

##### B. Observations on Network Characteristics

The above analysis provides a comprehensive solution to determine the traffic load, energy consumption, and network lifetime, as well as the energy hole boundary for a WSN. Based on these analytical results, we conclude two observations on network characteristics as follows.

(1) *If the sensor nodes are uniformly deployed in the network, the node density has no impact on the FNDT.* According to our analytical results, FNDT depends on maximum nodal energy consumption at  $S_0$ , while nodal energy consumption is determined by the traffic load of sensor nodes. According to Thm. 1, traffic load is unrelated to node density, which proves that node density has little impact on traffic load. It also indicates that it is useless to improve the lifetime by increasing the node density.

TABLE II  
PARAMETER SETTINGS

Parameters	Values
Initial energy of a sensor node $E_0$	0.5 J
Duty cycle $\gamma$	10%
Duration of a data period $T_r$	10 s
Energy consumption rate for idel listening $E_{idle}$	0.88 mJ/s
Data transmission rate $B$	512 Kb/s
Size of a data packet $\tau$	400 bits

(2) *There exists an optimal transmission range  $r$  to maximize the network lifetime.* According to Thm. 1, the transmission range of the sensor nodes  $r$  directly impacts the traffic load of sensor nodes, which determines the energy consumption and lifetime of the network. Therefore, we can set the optimal transmission range  $r$  for the sensor nodes to maximize the network lifetime. Since network lifetime can be estimated under a required death ratio  $i\%$  by Corollary 2, and the options of  $r$  are limited, the optimal  $r$  can be found to maximize  $l_{i\%}^i$  with brute-force testing [12], [17].

#### V. EXPERIMENTAL RESULTS

In this section, we validate our analytic results by extensive simulations in OMNET++ [5], [34]. We perform our simulations in various scenarios where a large number of sensors are deployed in a circular area with different network radii  $R$  and transmission ranges  $r$ . The sink is located at the centre of the network. We summarize the main parameter settings in Table II, and the settings of the energy consumption model are adopted from [12]. All of simulations are based on a collision-free MAC protocol without data loss to be consistent with our network model [6], [12], [30].

##### A. Comparison of Theoretical Analysis and simulation Results

1) *Traffic Load and Energy Consumption at  $S_0$ :* In Fig. 4(a), we compare the simulation results with the analytical results in terms of the traffic load at  $S_0$ . It shows that our analytic model is quite accurate in estimating the traffic load and the error rate between the theoretical results and simulation results is less than 5%, which might be qualified for most engineering applications. Fig. 4(b) shows the energy consumption comparison at  $S_0$ . As shown in this figure, our analytic model is also accurate in estimating the energy consumption of the sensor nodes. In addition, the transmission range of sensor node has a significant impact on energy consumption. Since the sensor nodes with maximum energy consumption die first in the network, it indicates that the first batch of dead nodes are not always closest to the sink.

2) *Traffic Load, Energy Consumption and Lifetime Comparison from  $S_1$  to  $S_{k-1}$ :* Fig. 5(a) shows the traffic load comparison between at  $S_0$  and the time when 5% sensor nodes die. Since the traffic load of the nodes far from the sink hardly changes, Fig. 5(a) shows the comparison among the nodes near the sink. In Fig. 5(a), the nodes forward more data after a few nodes die, because these nodes should undertake the data

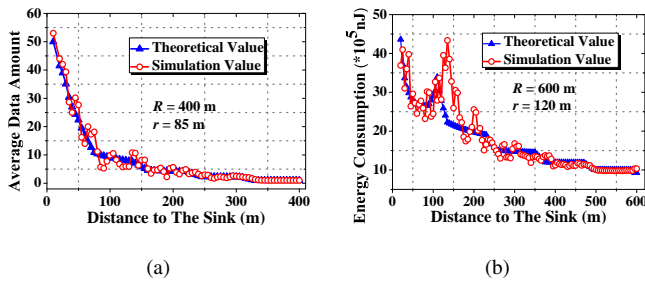


Fig. 4. (a) Traffic load at  $S_0$  ( $R = 400\text{m}$ ,  $r = 85\text{m}$ ); (b) Energy consumption at  $S_0$  ( $R = 600\text{m}$ ,  $r = 120\text{m}$ ).

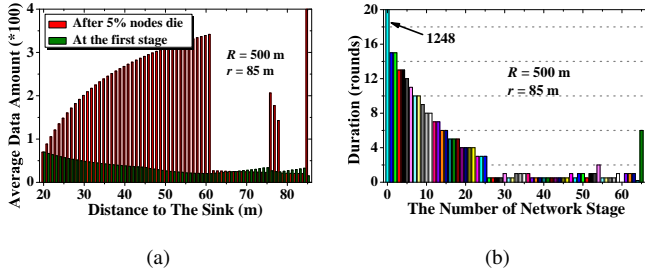


Fig. 5. (a) Traffic load comparison at  $S_0$  and after 5% sensor nodes die; (b) Duration comparison of different network stages.

original forwarded by the dead nodes. Thus, their nodal energy consumption rates increase, as known as funneling effect [20]. Fig. 5(b) shows the duration comparison of different stages. It can be seen that, except the lifetime  $l^{(0)}$  at stage  $S_0$  is 1248 rounds, the duration of other stages is much less than  $l^{(0)}$ . However, since the formation of energy hole needs to go through many stages, there is still a long period from FNDT to ANDT.

Fig. 6(a) and 6(b) show the nodal lifetime comparison in different network scenarios. The following phenomenon can be found. (a) The simulation results are consistent with theoretical analysis, and the maximum error of theoretical and simulation results is only 5.7%, which can meet the needs of general application. (b) The lifetime of each node in the figure actually shows death order of nodes and the formation process of the energy hole. The estimated results are consistent with the simulation results. (c) As shown in Fig. 6(a), the energy hole region is the region whose distance to the sink is less than  $r$ . However, if we increase the transmission range  $r$ , as shown in Fig. 6(b), the energy hole region changes to  $[38\text{m}, 155\text{m}]$ . This shows that the energy hole cannot be simply considered as near the sink and Alg. 2 can accurately estimate the location of the energy hole under different network parameters. Fig. 7 compares the ANDT estimation results between our analytical results and the analysis from [17]. [17] adopts an annuli-based analytic model to analyze the network lifetime, without considering the energy consumption for idle listening. It can be seen that our analytical results are more accurate than the results of [17]. However, with the increase of the network size, the gap between our model and [17] becomes small, which indicates the annuli-based analytic model in [17] are more applicable for large scale WSNs.

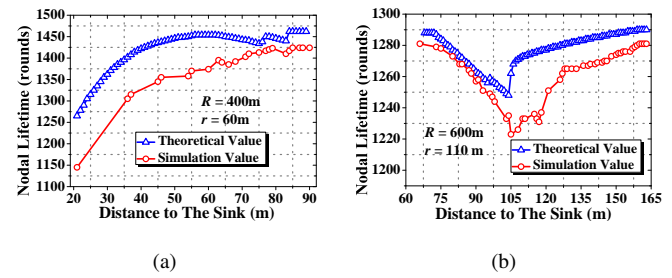


Fig. 6. (a) Nodal lifetime comparison ( $R = 400\text{m}$ ,  $r = 85\text{m}$ ); (b) Nodal lifetime comparison ( $R = 600\text{m}$ ,  $r = 110\text{m}$ ).

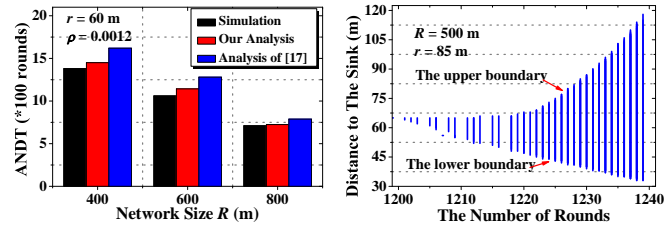


Fig. 7. ANDT comparison under different network size. Fig. 8. Energy hole evolution process.

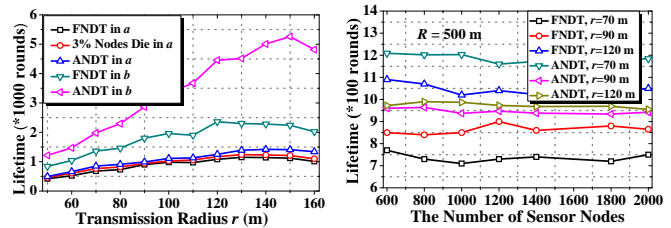


Fig. 9. Network lifetime comparison under different  $r^1$ . Fig. 10. Lifetime comparison under different node densities.

3) *Determination of Energy Hole Boundary*: The location of the energy hole can be obtained according to Alg. 2. Fig. 8 shows the change of the energy hole boundary during the network operation. In fact, the energy hole region can also be obtained from Fig. 6(a) and 6(b).

### B. Network Characteristic Observations

This section focuses on evaluating the network characteristics we have found in Section IV-B. Fig. 9 shows the lifetime of 0%(FNDT), 3% and 100%(ANDT) dead nodes ratio under different  $r$  and different energy consumption models. It is shown that there indeed exists an optimal  $r$  to maximize the network lifetime. Moreover, Fig. 9 shows that without the consideration of energy consumption for idle listening, the network ANDT is almost twice as FNDT. It also proves FNDT is only a part of the network life cycle. Fig. 10 shows network lifetime comparison under different node densities. It can be observed that node density has little impact on the network lifetime, including the FNDT and ANDT.

## VI. FURTHER DISCUSSIONS

In this section, we further discuss the significance of our analytical results and apply them to the WSN routing design

<sup>1</sup>Model  $a$  denotes the energy consumption model adopted in this paper, while Model  $b$  denotes the model without energy consumption in idle listening.

to mitigate energy hole problem and improve network lifetime.

Our analytical results and observations are instructive and useful for WSN deployment, design, and optimization. **(a)** It has been proposed that we can balance the energy consumption of the network by non-uniform node deployment [30], [35]. Since the energy hole boundary can be estimated, the location of the hotspot can also be derived to guide the non-uniform node deployment. Moreover, after the network deployment, we can re-evaluate the network lifetime by the proposed analytical model. **(b)** The paper presents a guideline for selecting optimal network parameters to improve network lifetime or performance. It has been demonstrated that different transmission radii lead to different network lifetime, which is useful to select an optimal  $r$  for a given network [5], [17]. Besides, since the energy consumption for idle listening is relatively large, it motivates us to design an energy-efficient sleep scheduling algorithm for the sensor nodes to further reduce the energy consumption and improve the network lifetime [33]. **(c)** Although geographic routing has inherent advantages to be applied into large scale WSNs, its negative influence on energy efficiency and network lifetime cannot be neglected. From our analysis, there is more than 80% energy left when the network is partitioned by the energy hole. Thus, our work should be helpful to provide navigation for designing an energy-efficient routing protocol.

In the following two sections, we intend to illustrate the significance of the proposed analytical results in guiding the WSN design and optimization. We take the routing design for instance to discuss how to improve the network lifetime, including both of FNDDT and ANDDT, by designing an energy efficient routing based on our analytical results.

#### A. Energy Efficient Routing Design based on Lifetime analytical results

According to our analytical results, since the nodes near the sink should forward the data from upstream nodes, the unbalanced energy consumption and energy hole problem cannot be avoided in a uniformly deployed data-gathering WSN [30]. However, it is still possible to mitigate the unbalanced energy consumption of the sensor nodes and improve the network lifetime by designing an energy-aware routing scheme. The main idea of most existing energy-aware routing solutions is to select the next hop based on the residual energy to avoid premature death in hotspot [32]. By this means, FNDDT can be significantly extended. However, it is observed from our analytical results that after the first node dies, the energy consumption rate of the substitute node increases sharply, leading to an accelerated ANDDT. Therefore, energy consumption balance should be considered from two aspects, nodal residual energy and energy consumption rate.

Since the cost function based routing has the inherent advantages in scalability and has been extensively studied for energy efficiency [34], our routing scheme concentrates on the cost function design. At first, an optimal energy cost function should map small changes in nodal residual energy to large changes in the value of the function. Such a function can rise sharply the cost of a routing path whose residual energy is low

and offset the cost reserving by path length reduction (if any exists), forcing nodes to select the route with more residual energy. Second, the energy consumption rate of nodes should be taken into consideration in cost function design. As the nodes in hotspots generally have higher energy consumption rate than other nodes, the energy can be further balanced with introducing this factor into the cost function. Based on the two principles, the **Double Cost Function based Routing (DCFR)** scheme can be designed as follows.

For the sensor node  $i$ , its neighboring sensor nodes whose distances to the sink are smaller than  $i$ 's constitute the candidate set of next hop, denoted by  $\{BN_i\}$ . For each sensor node  $j \in \{BN_i\}$ , denote the residual energy of node  $j$  by  $e_j^r$ , the energy consumption for transmitting a packet between  $i$  and  $j$  by  $e_{i,j}$ . Then, we define the energy cost  $c_{i,j}$  of the single hop between  $i$  and  $j$  as  $c_{i,j} = e_{i,j} \exp\left(1/\sin\left(\pi - \frac{\pi e_j^r}{2 e_0}\right)\right)$ .

Denote  $e_{t_x}^r$  and  $e_{t_y}^r$  as the residual energy of node  $j$  at  $t_x$  and  $t_y$  respectively. The energy consumption rate  $ER_j$  of node  $j$  is  $ER_j = \frac{e_{t_x}^r - e_{t_y}^r}{t_y - t_x}$ . Thus, the energy consumption rate cost  $rc_{i,j}$  from  $i$  to  $j$  is  $rc_{i,j} = e_{i,j} \exp\left(1/\sin\left(\frac{\pi}{2} + \frac{ER_j}{ER_{max}}\right)\right)$ , where  $ER_{max} = \max_{j \in \{BN_i\}}\{ER_j\}$ .

If we combine the energy cost and energy consumption rate cost, the total cost of node  $i$  selecting  $j$  as the next hop, denoted by  $TC_{i,j}$ , is

$$TC_{i,j} = c_{i,j} + rc_{i,j} = e_{i,j} \exp\left(1/\sin\left(\pi - \frac{\pi e_j^r}{2 e_0}\right)\right) + e_{i,j} \exp\left(1/\sin\left(\frac{\pi}{2} + \frac{ER_j}{ER_{max}}\right)\right). \quad (18)$$

Therefore, node  $i$  selects the node  $P$  with the smallest cost  $TC_{i,P}$  as the next hop, where  $P = \arg \min_{j \in BN_i}(TC_{i,j})$ . Note that, the cost function of each node can be calculated based on one hop neighbouring information. It indicates that the DCFR scheme is fully distributed and can be applied to large scale sensor networks.

#### B. Simulation Results on DCFR Scheme

In this section, we aim to validate the efficiency of the DCFR scheme via Omnet++ simulations. We compare it with two existing algorithms: Geographic Greedy Routing (GGR), and Distributed Energy Balanced Routing (DEBR) [10]. GGR is the routing scheme adopted in our previous analysis of this paper. Both DEBR and DCFR select the next hop based on the value of the cost function, while the former only considers the nodal residual energy. The parameter settings of the simulations are the same as the settings in Section V.

Fig. 11(a) compares the ANDDT with various network sizes. It illustrates that energy aware routing schemes, including DEBR and DCFR, have longer network lifetime than DC and GGR. With the consideration of nodal energy consumption rate, DCFR can achieve more balanced nodal energy consumption and a longer network lifetime than DEBR. Fig. 11(b) shows the changes of network lifetime with the increasing number of sensor nodes, under different routing algorithms. It

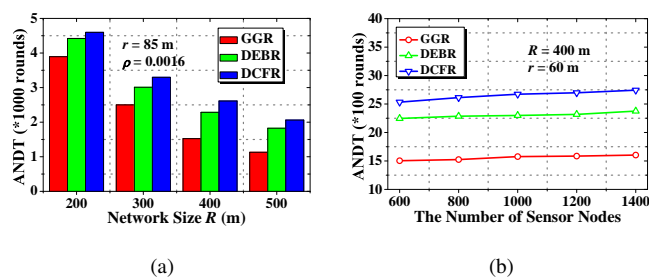


Fig. 11. (a) ANDT comparison under different routing schemes.; (b) ANDT comparison under different node densities.

can be seen that node density has little impact on the network lifetime under uniform node distribution, even in different routing algorithms.

## VII. CONCLUSION

In this paper, we have developed an analytic model to estimate the traffic load, energy consumption and lifetime of sensor nodes in a data-gathering WSN. With the analytic model, we have calculated the network lifetime under a given percentage of dead nodes, and analyzed the emerging time and location of energy hole, as well as its evolution process. Moreover, two network characteristics have been found based on our analytic results, which can be leveraged to guide the WSN design and optimization. Our simulation results demonstrate that the proposed analytic model can estimate the network lifetime and energy hole evolution process within an error rate smaller than 5%. Finally, we have applied our analytic results to WSN routing. The improved routing scheme based on our analytical results can efficiently balance the energy consumption and prolong the network lifetime. In our future work, we will extend the lifetime analysis into energy harvesting WSNs. Since sensor nodes are supplied by stochastic renewable energy, it is very challenging to analyze and optimize the network lifetime under the continuous and unstable energy supply.

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